1 General/Finance/Statistics

Program Library

Percentage
Metric System
Memory
Games
Dates
Finance
Mortgages
Statistics

Sinclair Cambridge Programmable

Hobsons Press (Cambridge) Ltd

General / Finance / Statistics

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CONTENTS

How to use these programs

Each program is arranged as follows:

1. On the left of the page, explanatory information and the 'execution sequence', the sequence of keystrokes necessary for running the program. Results displayed are printed in gold.
2. In the first column on the right hand side of the page, the sequence of keystrokes which make up the program.
3. In the second and third columns on the right hand side of the page, the program in check symbol and step number form (see section on checking the program).

Notes

1. Where a key has more than one function, the relevant function is printed as the keystroke in the first column
   e.g. the keystroke [8] (cos / arccos) may appear as 8, cos or arccos.
2. The symbol ▾ within a program always refers to the key [·/EE/–] (▾)
3. The symbol # refers to [3] (ChN/#)
4. The abbreviation gin is 'go if neg' and so refers to the key [1] (go if neg)

Entering the program

To enter a program into the calculator:

1. Press [▲▼] [▲▼] [2] (go to) [0] [0]	Display shows step programmed at 00 in check symbol form as described below.
2. Press [▲▼] [RUN] (learn)	No change in display.
3. Press the sequence of keys for the program as shown in the first column of the program page.	At each stage the step about to be overwritten is displayed. When the machine is first switched on every step is zero.
4. Press [C/CE]	Normal number display is resumed.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]	The step programmed at 00 will be displayed.

Checking the program

Each of the programs in the library is shown in check symbol form in the second column on the right-hand side of the page.

Press [▲▼] [C/CE] (step) repeatedly, and at each stage the check symbol will appear on the left of the display with the step number on the right. Ignore the four zeros in the display.

e.g. A.0000 03
(check symbol) (step number)

After stepping through the program, press

[▲▼] [▲▼] [2] (go to) [0] [0] before execution.

Finally, press [C/CE] and the program is ready for use.

Correcting the program

If the check symbol for a particular step number is not as indicated in the last two columns of the program page:

1. Press [▲▼] [▲▼] [2] (go to)
   followed by the step number if the appropriate step number is not already displayed.
2. Press [▲▼] [RUN] (learn)
3. Enter the correct keystroke. The display will then show the next step in the program. If this is also incorrect, enter the correct keystroke. At each stage, the step about to be overwritten will be displayed.
4. When correction has been completed, press [C/CE]. Any step which has not been overwritten will not be affected.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]

Note
To restore normal use of the calculator after entering or checking the program, press [C/CE]

Running the program

Press the sequence of keys as shown in the program library in the execution sequence. Results displayed are printed in gold.

POWERS

ln	4	00
X	·	01
stop	0	02
=	—	03
▼	A	04
e^x	4	05
stop	0	06
▼	A	07
goto	2	08
0	0	09
0	0	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
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		27
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		31
		32
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		34
		35

To find x^y

Execution:

x / RUN / y / RUN / x^y $\qquad x > 0$

This program can be used inside parentheses and does not affect memory.

ROOTS

ln	4	00
÷	G	01
stop	0	02
=	—	03
▼	A	04
e^x	4	05
stop	0	06
▼	A	07
goto	2	08
0	0	09
0	0	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
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		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

To find the yth root of x

Execution:

x / RUN / y / RUN / $\sqrt[y]{x}$

PERCENTAGE FUNCTIONS

(	6	00
X	·	01
stop	0	02
÷	G	03
#	3	04
1	1	05
0	0	06
0	0	07
=	–	08
)	6	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Execution:

1. x / = / RUN / a / RUN / a% of x
2. / x / + / RUN / a / RUN / a% of x / = / x + a% of x
3. / x / – / RUN / a / RUN / a% of x / = / x – a% of x /

MEMORY FUNCTIONS

▼	A	00
MEx	5	01
stop	0	02
rcl	5	03
=	–	04
▼	A	05
MEx	5	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
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		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Memory contains y initially:

Execution:

M+: x / RUN / + / RUN / (x in display, x + y in memory)

M–: x / RUN / – / RUN / (x in display, y – x in memory)

M×: x / RUN / × / RUN / (x in display, xy in memory)

M÷: x / RUN / ÷ / RUN / (x in display, y ÷ x in memory)

MC: x / RUN / C/CE / C/CE / × / RUN / (x in display, 0 in memory)

STO–: x / RUN / C/CE / C/CE / – / RUN / (x in display, –x in memory)

In each case, the original contents y of the memory are displayed after the first / RUN /.

HOLDING AN EXTRA CONSTANT IN PROGRAM MEMORY

Suppose there is an extra number you want to store while doing calculations, for example the velocity of light

c = 2·997925 x 10^8 m s^{-1}.

The number can be stored in the program memory as shown opposite.

Each time you need to use the constant, just press / RUN /. This will enter the constant and complete the last operation, just like the sequence / ▲▼ / rcl / = / if the constant were stored in the memory. However, the memory can still be used to store other numbers, and the program will also operate inside parentheses.

This idea can be extended to store several constants if required.

#	3	00
2	2	01
·	A	02
9	9	03
9	9	04
7	7	05
9	9	06
2	2	07
5	5	08
·	A	09
8	8	10
=	—	11
stop	0	12
▼	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

HOLDING TWO EXTRA CONSTANTS IN PROGRAM MEMORY

The exact way this is done depends on the way that the constants will be needed.

1. One constant readily accessible, the other a little more difficult to recover

To use the const. 1·0748321 just press / RUN /

To use the const. 4·386579 press ▲▼ / ▲▼ / goto / 1 / 6 / RUN

This program can be used inside parentheses and does not affect normal memory use.

#	3	00
1	1	01
·	A	02
0	0	03
7	7	04
4	4	05
8	8	06
3	3	07
2	2	08
1	1	09
=	—	10
stop	0	11
▼	A	12
goto	2	13
0	0	14
0	0	15
#	3	16
4	4	17
·	A	18
3	3	19
8	8	20
6	6	21
5	5	22
7	7	23
9	9	24
=	—	25
stop	0	26
▼	A	27
goto	2	28
0	0	29
0	0	30
		31
		32
		33
		34
		35

HOLDING TWO EXTRA CONSTANTS IN PROGRAM MEMORY

2. Constants wanted alternately

#	3	00
1	1	01
·	A	02
0	0	03
7	7	04
4	4	05
8	8	06
3	3	07
2	2	08
1	1	09
=	–	10
stop	0	11
#	3	12
4	4	13
·	A	14
3	3	15
8	8	16
6	6	17
5	5	18
7	7	19
9	9	20
=	–	21
stop	0	22
▼	A	23
goto	2	24
0	0	25
0	0	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Pressing / RUN / will recall constants alternately.

To recover a constant out of turn press
▲▼ / ▲▼ / goto / 0 / 0 / RUN / for 1·0748321
and
▲▼ / ▲▼ / goto / 1 / 2 / RUN / for 4·386579

(If the second constant is wanted at the beginning of a calculation then / RUN / RUN / will work too.)

This program can be used inside parentheses and does not affect normal memory use.

HOLDING TWO EXTRA CONSTANTS IN PROGRAM MEMORY

3. Either constant to be used repeatedly

#	3	00
1	1	01
·	A	02
0	0	03
7	7	04
4	4	05
8	8	06
3	3	07
2	2	08
1	1	09
=	–	10
stop	0	11
▼	A	12
goto	2	13
0	0	14
0	0	15
#	3	16
4	4	17
·	A	18
3	3	19
8	8	20
6	6	21
5	5	22
7	7	23
9	9	24
=	–	25
stop	0	26
▼	A	27
goto	2	28
1	1	29
6	6	30
		31
		32
		33
		34
		35

Operation:

/ RUN / recalls first constant whenever needed until first recall of second constant.

For first recall of second constant:
▲▼ / ▲▼ / goto / 1 / 6 / RUN /
Subsequent / RUN / will recall second constant.

To recall first constant again press
▲▼ / ▲▼ / goto / 0 / 0 / RUN /

STORING THREE OR MORE CONSTANTS IN PROGRAM MEMORY

As an example, three important physical constants which are often associated are stored in the program opposite, namely:

T_0 = absolute temperature of 0°C = 273·152K

k = Boltzmann's constant
= $1{\cdot}380622 \times 10^{-23}$ J K^{-1}

q = electronic charge = $1{\cdot}6021917 \times 10^{-19}$C

For example, to calculate the current in a diode from

$$I = I_s\left(\exp\left(\frac{qV}{kT}\right) - 1\right)$$

where V is the applied voltage, T the junction temperature and I_s the saturation current, use pre-execution:

/ ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

T / + / RUN / X / RUN / ÷ / RUN / ÷ / X / V / = / ▲▼ / ▲▼ / e^x / – / 1 / X / I_s / = / I
with T in °C and V in volts.

For repeated execution, I_s could be stored in memory.

#	3	00
2	2	01
7	7	02
3	3	03
·	A	04
1	1	05
5	5	06
=	–	07
stop	0	08
#	3	09
1	1	10
·	A	11
3	3	12
8	8	13
0	0	14
6	6	15
2	2	16
·	A	17
·	A	18
2	2	19
3	3	20
=	–	21
stop	0	22
#	3	23
1	1	24
·	A	25
6	6	26
0	0	27
2	2	28
2	2	29
·	A	30
·	A	31
1	1	32
9	9	33
=	–	34
stop	0	35

The constants can be recalled out of order by using the pre-execution:

/ ▲▼ / ▲▼ / goto / 0 / 9 / for k

/ ▲▼ / ▲▼ / goto / 2 / 3 / for q

or

/ ▲▼ / ▲▼ / goto / 0 / 0 / for T_0

This idea can be adapted to store three 9-digit numbers, four 6-digit numbers, five 4-digit numbers, etc., the decimal point counting as a digit. Use / = / steps to fill the remaining spaces, or / ▼ / goto / 0 / 0 / etc. if there is room.

LOGARITHMS TO BASE A

If base is not to be kept the same

ln	4	00
sto	2	01
stop	0	02
ln	4	03
÷	G	04
rcl	5	05
=	–	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
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		26
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		28
		29
		30
		31
		32
		33
		34
		35

Execution:

a / RUN / x / RUN / $\log_a x$

LOGARITHMS TO BASE A

If the same base is to be used repeatedly

ln	4	00
sto	2	01
stop	0	02
ln	4	03
÷	G	04
rcl	5	05
=	–	06
▼	A	07
goto	2	08
0	0	09
2	2	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
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		31
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		35

If the same base is to be used repeatedly

Execution:

a / RUN / x_1 / RUN / $\log_a x_1$ / x_2 / RUN / $\log_a x_2$ / · · ·

To set a new base:

▲▼ / ▲▼ / goto / 0 / 0 / a' / RUN / · · · etc.

CONVERSIONS

Degrees Fahrenheit to degrees Centigrade

—	F	00
#	3	01
3	3	02
2	2	03
÷	G	04
#	3	05
1	1	06
·	A	07
8	8	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
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		30
		31
		32
		33
		34
		35

Execution:

°F / RUN / °C

CONVERSIONS

Degrees Centigrade to degrees Fahrenheit

X	·	00
#	3	01
1	1	02
·	A	03
8	8	04
+	E	05
#	3	06
3	3	07
2	2	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Execution:

°C / RUN / °F

CONVERSIONS

Feet and inches to metres

X	·	00
#	3	01
1	1	02
2	2	03
+	E	04
stop	0	05
X	·	06
#	3	07
·	A	08
0	0	09
2	2	10
5	5	11
4	4	12
=	–	13
stop	0	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Execution:

feet / RUN / inches / RUN / metres

Note: 0 must be entered if 0 inches.

CONVERSIONS

Metres to feet and inches

÷	G	00
#	3	01
·	A	02
3	3	03
0	0	04
4	4	05
8	8	06
–	F	07
(	6	08
–	F	09
#	3	10
1	1	11
=	–	12
▼	A	13
gin	1	14
2	2	15
1	1	16
▼	A	17
goto	2	18
0	0	19
9	9	20
+	E	21
#	3	22
1	1	23
=	–	24
sto	2	25
)	6	26
=	–	27
stop	0	28
rcl	5	29
X	·	30
#	3	31
1	1	32
2	2	33
=	–	34
stop	0	35

Execution:

metres / RUN / feet / RUN / inches

Note: This program may take some time to execute.

CONVERSIONS

Pounds and ounces to kilograms

Execution:

lb / RUN / oz / RUN / kg

Note: Enter 0 if 0 oz

+	E	00
+	E	01
+	E	02
+	E	03
+	E	04
stop	0	05
÷	G	06
#	3	07
3	3	08
5	5	09
·	A	10
2	2	11
7	7	12
4	4	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CONVERSIONS

Kilograms to pounds and ounces

Execution:

kg / RUN / lb / RUN / oz

÷	G	00
#	3	01
·	A	02
4	4	03
5	5	04
3	3	05
6	6	06
—	F	07
(	6	08
—	F	09
#	3	10
1	1	11
=	—	12
▼	A	13
gin	1	14
2	2	15
1	1	16
▼	A	17
goto	2	18
0	0	19
9	9	20
+	E	21
#	3	22
1	1	23
=	—	24
sto	2	25
)	6	26
=	—	27
stop	0	28
rcl	5	29
+	E	30
+	E	31
+	E	32
+	E	33
=	—	34
stop	0	35

CONVERSIONS

Degrees, minutes, seconds to decimal degrees
Hours, minutes, seconds to decimal hours

+	E	00
(	6	01
stop	0	02
X	·	03
#	3	04
6	6	05
0	0	06
+	E	07
stop	0	08
÷	G	09
#	3	10
3	3	11
6	6	12
0	0	13
0	0	14
=	–	15
)	6	16
=	–	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Execution:

deg / RUN / min / RUN / sec / RUN / decimal degrees

or

hr / RUN / min / RUN / sec / RUN / decimal hr

Note: Min and sec must be entered as 0 if zero.

CONVERSIONS

Decimal degrees to degrees, minutes and seconds
Decimal hours to hours, minutes and seconds
Decimal minutes to minutes and seconds

–	F	00
(	6	01
–	F	02
#	3	03
1	1	04
=	–	05
▼	A	06
gin	1	07
1	1	08
4	4	09
▼	A	10
goto	2	11
0	0	12
2	2	13
+	E	14
#	3	15
1	1	16
=	–	17
sto	2	18
)	6	19
=	–	20
stop	0	21
rcl	5	22
X	·	23
#	3	24
6	6	25
0	0	26
=	–	27
stop	0	28
▼	A	29
goto	2	30
0	0	31
0	0	32
		33
		34
		35

Execution:

(i) degrees as decimal / RUN / D / RUN / RUN / M / RUN / S

(ii) hours as decimal / RUN / hours / RUN / RUN / mins / RUN / secs

(iii) minutes as decimal / RUN / mins / RUN / secs

The number of seconds will be shown as a decimal. To use the program again, just enter the new number of degrees, hours or minutes.

In (i) and (ii), after the second RUN the display shows the number of minutes as decimal.

MATCHSTICK GAME

You put N matchsticks down on the table. At each turn, each player may pick up 1, 2, or 3 matchsticks; because you choose the starting number N, the machine has the first turn. The object of the game is to avoid picking up the last matchstick; thus if either player leaves 1 matchstick after his turn he has won.

Execution:

N / RUN / machine plays

/ 1, 2 or 3 / RUN / you play

/ RUN / machine plays

/ 1, 2 or 3 / RUN / you play

etc.

Display each time shows number of matchsticks remaining.

sto	2	00
–	F	01
(	6	02
rcl	5	03
+	E	04
#	3	05
3	3	06
–	F	07
#	3	08
4	4	09
–	F	10
–	F	11
▼	A	12
gin	1	13
0	0	14
7	7	15
+	E	16
▼	A	17
gin	1	18
2	2	19
4	4	20
#	3	21
5	5	22
–	F	23
#	3	24
4	4	25
=	–	26
)	6	27
–	F	28
stop	0	29
=	–	30
stop	0	31
=	–	32
=	–	33
=	–	34
=	–	35

PSEUDO–RANDOM DICE THROWER

This dice is slightly biased, but not too heavily to be convincing!

Execution:

Choose any starting value x between 0 and 1.

x / RUN / d_1 / RUN / d_2 / RUN / d_3 / etc.

where d_1, d_2, d_3 are successive 'throws'.

X	·	00
#	3	01
1	1	02
0	0	03
1	1	04
÷	G	05
#	3	06
1	1	07
7	7	08
+	E	09
(	6	10
–	F	11
+	E	12
#	3	13
1	1	14
=	–	15
▼	A	16
gin	1	17
1	1	18
2	2	19
sto	2	20
)	6	21
=	–	22
stop	0	23
rcl	5	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

MOON LANDING GAME

The object of the moon landing game is to land the Lunar Module (LEM) safely on the moon's surface.

The LEM's rocket motor has 'bang-bang' control; in other words it can either be on ('burn') or off ('coast'). Thus the landing consists of a series of burns and coasts of various lengths. Your job is to choose the lengths of these stages. You are of course limited by the amount of fuel on board.

For convenience in programming, the landing is modelled by two programs.

The first program models the first long burn which gets the LEM out of lunar orbit and slows it to a near-vertical descent above the landing site.

The second program models the subsequent series of coasts and burns which should slow the LEM to a soft landing on the moon.

The LEM can withstand landing speeds of up to 5 metres per second. Speeds above this may cause spectacularly disastrous results!

The equations used in the programs are of course only approximate, but the approximations can all be justified.

MOON LANDING GAME

Getting out of orbit

This program computes the final speed, amount of fuel remaining and height after the long initial 'burn'. The initial mass of the LEM, M_0 is 3000kg, including fuel mass F_0 = 2000kg. Orbital speed is 1·7km s^{-1} in close lunar orbit at a height H_0 chosen by the pilot – we suggest 25 to 50km. The rocket motor burns 2kg of fuel per second with an exhaust velocity of 2400m s^{-1}, giving a thrust of 4800N.

The final speed V_1, height H_1, mass M_1 and fuel left F_1 are modelled by:

$$V_1 = V_0 + 2400 \ln\left(\frac{M_0 - 2T}{M_0}\right) \quad \text{m s}^{-1}$$

$$H_1 = \frac{H_0}{2} \quad \text{m}$$

$$F_1 = F_0 - 2T \quad \text{kg}$$

$$M_1 = M_0 - 2T \quad \text{kg}$$

'Burn' time left is given by

$$T_1 = T_0 - T \quad \text{s} \qquad \text{where } T_0 = \frac{F_0}{2} \quad \text{s}$$

Execution:

Choose T and H_0

/ RUN / T / RUN / F_1 / RUN / V_1

H_0 / ÷ / 2 / = / H_1

Try different values of T if you wish.

The results from this program are used as starting values for the vertical descent phase.

#	3	00
1	1	01
0	0	02
0	0	03
0	0	04
–	F	05
sto	2	06
stop	0	07
+	E	08
+	E	09
stop	0	10
rcl	5	11
÷	G	12
rcl	5	13
÷	G	14
#	3	15
3	3	16
=	–	17
ln	4	18
X	·	19
#	3	20
2	2	21
4	4	22
0	0	23
0	0	24
+	E	25
#	3	26
1	1	27
7	7	28
0	0	29
0	0	30
=	–	31
stop	0	32
=	–	33
=	–	34
=	–	35

MOON LANDING GAME –

Vertical descent

The exact equations of motion during the vertical descent are modelled by linear approximations using the equations below:

'Burn'

$$F_{i+1} = F_i - 2T_b$$

$$V_{i+1} = V_i + 1{\cdot}6T_b - \frac{4800}{M_{av}} T_b$$

$$H_{i+1} = H_i - V_{av} T_b$$

$$T_{i+1} = T_i - T_b$$

'Coast'

$$F_{i+1} = F_i$$

$$V_{i+1} = V_i + 1{\cdot}6T_c$$

$$H_{i+1} = H_i - V_{av} T_c$$

$$T_{i+1} = T_i$$

where $M_{av} = M_i - T = \frac{M_i + M_{i+1}}{2}$

and $V_{av} = \frac{V_i + V_{i+1}}{2}$

The 'coast' equations are exact, but the 'burn' approximations are less accurate for 'burn' times longer than about 45 seconds. Either choose a succession of shorter 'burn' times or correct V_{i+1} and H_{i+1} as below:

$$V'_{i+1} = V_{i+1} - \frac{400T_b}{F_i + 1000}$$

$$H'_{i+1} = H_{i+1} - \frac{400T_b^2}{F_i + 1000}$$

sto	2	00
+	E	01
stop	0	02
+	E	03
stop	0	04
#	3	05
1	1	06
0	0	07
0	0	08
0	0	09
+	E	10
rcl	5	11
÷	G	12
–	F	13
X	·	14
#	3	15
2	2	16
4	4	17
0	0	18
0	0	19
+	E	20
#	3	21
·	A	22
8	8	23
X	·	24
rcl	5	25
–	F	26
(	6	27
+	E	28
+	E	29
stop	0	30
)	6	31
stop	0	32
X	·	33
rcl	5	34
stop	0	35

Execution:

Decide whether to 'burn' or 'coast' and for how long (T_b or T_c seconds)

Burn: T_i / – / T_b / RUN / T_{i+1} / RUN / F_{i+1} / RUN / V_i / RUN / V_{i+1} / RUN / + / H_i / = / H_{i+1}

Coast: T_c / ▲▼ / sto / ▲▼ / ▲▼ / goto / 2 / 1 / RUN / V_i / RUN / V_{i+1} / RUN / + / H_i / = / H_{i+1}

Tabulate the results as below:

Burn	Coast	Time T_i	Fuel F_i	Speed V_1	Height H_i
*700		300	600	191·348	15000
	20	300	600	223·348	10853·04
10		290	580	209·15933	8690·504
	20	290	580	241·15933	4187·3174
11		279	558	225·10733	1622·8508
	7	279	558	236·30733	7·8995

You are now 7·9 metres above the moon travelling at 236 metres per sec. Crash!!! Better luck next time!

*using 'getting out of orbit' program.

SUNDAY LETTER 1900 – 2099

Execution:

year / RUN / result

Result	Sunday letter
1	A
2	B
3	C
4	D
5	E
6	F
7	G

To find Easter 1900–2099

Use this program to find the Sunday letter and also find the Golden Number.

Locate the Golden Number in the first column of the Table and read across to find the date of the Paschal Full Moon in the second column.

Read down the third column from the day following the Paschal Full Moon to find the Sunday letter. The date opposite this letter in column 2 is the date of Easter Sunday.

e.g. 1976 Golden number = 1
Sunday letter = C

Column 1 gives Paschal Full Moon as April 14. First C below April 14 is April 18.

Therefore April 18 = Easter Sunday.

—	F	00
#	3	01
2	2	02
1	1	03
0	0	04
7	7	05
÷	G	06
#	3	07
·	A	08
8	8	09
+	E	10
#	3	11
7	7	12
+	E	13
▼	A	14
gin	1	15
1	1	16
1	1	17
(	6	18
—	F	19
+	E	20
#	3	21
1	1	22
=	—	23
▼	A	24
gin	1	25
2	2	26
0	0	27
)	6	28
—	F	29
+	E	30
#	3	31
8	8	32
=	—	33
stop	0	34
=	—	35

GOLDEN NUMBER 1900 – 2099

Execution:

year / RUN / Golden number

Table to find Easter 1900–2099

Golden number	Day and month	Sunday letter
—	March 21	C
14	22	D
3	23	E
—	24	F
11	25	G
—	26	A
19	27	B
8	28	C
—	29	D
16	30	E
5	31	F
—	April 1	G
13	2	A
2	3	B
—	4	C
10	5	D
—	6	E
18	7	F
7	8	G
—	9	A
15	10	B
4	11	C
—	12	D
12	13	E
1	14	F
—	15	G
9	16	A
17	17	B
6	18	C
—	19	D
—	20	E
—	21	F
—	22	G
—	23	A
—	24	B
—	25	C

—	F	00
#	3	01
1	1	02
9	9	03
0	0	04
0	0	05
—	F	06
#	3	07
1	1	08
9	9	09
=	—	10
▼	A	11
gin	1	12
1	1	13
9	9	14
▼	A	15
goto	2	16
0	0	17
6	6	18
+	E	19
#	3	20
2	2	21
0	0	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

DAY OF THE WEEK OF CHRISTMAS DAY (1900 – 2099)

Execution:

year (in full) / RUN / day as a number

where 1 = Sunday
2 = Monday, etc

X	·	00
#	3	01
1	1	02
·	A	03
2	2	04
4	4	05
9	9	06
6	6	07
—	F	08
#	3	09
2	2	10
6	6	11
3	3	12
1	1	13
+	E	14
#	3	15
7	7	16
+	E	17
▼	A	18
gin	1	19
1	1	20
5	5	21
(	6	22
—	F	23
+	E	24
#	3	25
1	1	26
=	—	27
▼	A	28
gin	1	29
2	2	30
4	4	31
)	6	32
=	—	33
stop	0	34
=	—	35

		00
		01
		02
		03
		04
		05
		06
		07
		08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

DISCOUNT

Discounts a series of prices by a given percentage.

Execution:

percentage discount / RUN / gross price / RUN / discounted price / gross price / RUN / discounted price /

To enter a new discount:

▲▼ / ▲▼ / goto / 0 / 0 / new discount / RUN /

Example:

I want to reduce all the prices in my shop by 9% for the January sale. Items cost £1·35, £0·76, etc.

Enter discount [9] [RUN]

Gross price [1] [·] [3] [5] [RUN]

Display shows discounted price £1·23

Gross price [0] [·] [7] [6] [RUN]

Display shows discounted price 69p etc.

(Results shown on display have been rounded to nearest penny.)

—	F	00
÷	G	01
#	3	02
1	1	03
0	0	04
0	0	05
+	E	06
#	3	07
1	1	08
=	—	09
sto	2	10
stop	0	11
X	·	12
rcl	5	13
=	—	14
▼	A	15
goto	2	16
1	1	17
1	1	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

MARK—UP

Marks up a series of prices by a given percentage.

Execution:

percentage mark-up / RUN / price / RUN / marked up price / another price / RUN / marked up price / etc.

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
=	—	08
sto	2	09
stop	0	10
X	·	11
rcl	5	12
=	—	13
▼	A	14
goto	2	15
1	1	16
0	0	17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

MARK—UP, GROSS PERCENTAGE INCREASE GIVEN

Marks up prices by a given percentage of their *new* value. Thus £90 marked up by 10% will give £100; the increase of £10 is 10% of the gross price £100.

Execution:

percentage / RUN / old price / RUN / new price / another old price / RUN / new price / etc.

To enter a new percentage:

▲▼ / ▲▼ / goto / 0 / 0 / new percentage / RUN / old price / etc.

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
—	F	05
#	3	06
1	1	07
—	F	08
÷	G	09
=	—	10
sto	2	11
stop	0	12
X	·	13
rcl	5	14
=	—	15
▼	A	16
goto	2	17
1	1	18
2	2	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

DISCOUNT OR TAX, PERCENTAGE OF NET SUM GIVEN

Example:

VAT is at 8%. I price my goods VAT inclusive and wish to work out their net prices.

Execution:

percentage / RUN / gross price / RUN / deduction or tax / RUN / net price / another gross price / RUN / deduction or tax / RUN / net price / etc.

To enter a new percentage:

/ C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0 / new percentage / etc.

÷	G	00
(	6	01
+	E	02
#	3	03
1	1	04
0	0	05
0	0	06
=	—	07
)	6	08
=	—	09
sto	2	10
stop	0	11
—	F	12
(	6	13
X	·	14
rcl	5	15
)	6	16
stop	0	17
=	—	18
▼	A	19
goto	2	20
1	1	21
1	1	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

PERCENTAGE CHANGE ARISING FROM MARK—UP OR DISCOUNT CHANGE

Example:

VAT is cut from 25% to 12½%. What percentage difference does this make? (By what percentage should prices be cut?)

Execution:

old mark-up / RUN / new mark-up / RUN / percentage change

Enter discounts as negative mark-ups.

Solution to example:

Old mark-up [2] [5] [RUN]

New mark-up [1] [2] [·] [5] [RUN]

Percentage change = —10%, i.e. 10% decrease.

sto	2	00
÷	G	01
#	3	02
1	1	03
0	0	04
0	0	05
+	E	06
#	3	07
1	1	08
÷	G	09
X	·	10
(	6	11
stop	0	12
—	F	13
rcl	5	14
)	6	15
=	—	16
stop	0	17
▼	A	18
goto	2	19
0	0	20
0	0	21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

MORTGAGE REPAYMENTS

Given:

Amount of mortgage

Length of mortgage

Rate of interest

Finds:

Monthly repayment

Execution:

rate / RUN / term / RUN / amount / RUN / repayment

Example 1:

My mortgage is for a sum of £8500 at 10¾% over 25 years. What must I pay each month?

Rate [1] [0] [·] [7] [5] [RUN]

Term [2] [5] [RUN]

Amount [8] [5] [0] [0] [RUN]

Monthly repayment = £82·58

Example 2:

My mortgage has 12 years to run. The present balance is £4270. The rate of interest has just been increased to 11%. How much will my new monthly repayment be?

Rate [1] [1] [RUN]

Term [1] [2] [RUN]

Amount [4] [2] [7] [0] [RUN]

My new monthly payment is £54·81

Note: If you want to work out what your new monthly payment will be following a change of interest rate, and you do not know what your balance is, use one of the programs on page 44 or 45 to calculate your present balance.

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
sto	2	06
#	3	07
1	1	08
=	—	09
ln	4	10
X	·	11
stop	0	12
=	—	13
▼	A	14
e^x	4	15
÷	G	16
—	F	17
#	3	18
1	1	19
—	F	20
÷	G	21
rcl	5	22
÷	G	23
stop	0	24
÷	G	25
÷	G	26
#	3	27
1	1	28
2	2	29
=	—	30
stop	0	31
▼	A	32
goto	2	33
0	0	34
0	0	35

BALANCE OUTSTANDING ON A MORTGAGE

Given:

Amount of original mortgage

Monthly repayment

Number of years since mortgage was originally taken out

Rate of interest

Finds:

Balance

Execution:

rate / RUN / number of years / RUN / monthly repayment / RUN / original amount / RUN / balance

Example:

I bought a house seven years ago and took out a mortgage for £5500 at 11½% interest. My monthly repayment has been £70. I now want to sell my house and pay off the mortgage. How much will I have to pay?

Rate [1] [1] [·] [5] [RUN]

Number of years [7] [RUN]

Monthly payment [7] [0] [RUN]

Original amount [5] [5] [0] [0] [RUN]

Balance = £3438

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
=	–	05
sto	2	06
+	E	07
#	3	08
1	1	09
=	–	10
ln	4	11
X	·	12
stop	0	13
=	–	14
▼	A	15
e^x	4	16
X	·	17
(	6	18
stop	0	19
X	·	20
#	3	21
1	1	22
2	2	23
÷	G	24
rcl	5	25
=	–	26
sto	2	27
–	F	28
+	E	29
stop	0	30
)	6	31
+	E	32
rcl	5	33
=	–	34
stop	0	35

BALANCE OUTSTANDING ON A MORTGAGE

Given:

Monthly repayments

Present rate of interest

Number of years mortgage has to run

Finds:

Balance outstanding

This program is useful for finding the balance outstanding when the interest rate and/or repayment has changed since the beginning of the mortgage, but the number of years to run is known.

Execution:

interest rate / RUN / number of years to run / RUN / monthly payment / RUN / balance

Example:

My mortgage has 12 years to run. My present monthly payment is £50 and the interest rate is 10½%. What is the outstanding balance?

Rate [1] [0] [·] [5] [RUN]

Years to run [1] [2] [RUN]

Monthly payment [5] [0] [RUN]

Balance = £3990 to nearest pound.

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
sto	2	06
#	3	07
1	1	08
=	–	09
ln	4	10
X	·	11
stop	0	12
–	F	13
=	–	14
▼	A	15
e^x	4	16
–	F	17
#	3	18
1	1	19
–	F	20
X	·	21
stop	0	22
X	·	23
#	3	24
1	1	25
2	2	26
÷	G	27
rcl	5	28
=	–	29
stop	0	30
=	–	31
=	–	32
=	–	33
=	–	34
=	–	35

MORTGAGE TERM

Given:

Amount of mortgage
Monthly payment
Rate of interest

Finds:

Term of mortgage in years

Execution:

rate / RUN / amount of mortgage / RUN / monthly payment / RUN / term

Example 1:

I wish to take out a £7000 mortgage at 11% interest. I can afford to repay £80 per month. What is the shortest term mortgage I can have?

Rate [1] [1] [RUN]
Amount of mortgage [7] [0] [0] [0] [RUN]
Repayment [8] [0] [RUN]

Result is 15·52 years, so in practice I would take out a 15 years mortgage, with a monthly repayment of £81·12 (calculated using the program on page 43).

Example 2:

The balance on my mortgage is £5100 and my monthly repayment is £55. I have just been informed that the interest rate has been increased to 11¼%. I cannot afford a higher repayment and so I shall have to extend the term of the mortgage. When will the mortgage be paid off?

Rate [1] [1] [·] [2] [5]
Amount of mortgage [5] [1] [0] [0]
Repayment [5] [5]

Result is 19·085

So the new term is 19 years with a small balance payable at the end.

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
X	·	05
sto	2	06
stop	0	07
÷	G	08
stop	0	09
÷	G	10
#	3	11
1	1	12
2	2	13
—	F	14
#	3	15
1	1	16
—	F	17
÷	G	18
=	—	19
ln	4	20
÷	G	21
(	6	22
rcl	5	23
+	E	24
#	3	25
1	1	26
=	—	27
ln	4	28
)	6	29
=	—	30
stop	0	31
=	—	32
=	—	33
=	—	34
=	—	35

TAX RELIEF ON A MORTGAGE

Given:

Balance of mortgage
Interest rate

Finds:

Annual tax relief (for standard rate taxpayers)

Execution:

balance / RUN / interest rate / RUN / tax relief

Example:

My mortgage balance is £6000 and the rate of interest is 10¾%. How much tax will I save this year?

Balance [6] [0] [0] [0]
Rate [1] [0] [·] [7] [5]

Tax relief = £225·75

Note: This program assumes tax rate of 35p in the pound. Should this change, the figures in steps 07 and 08 should be altered to correspond.

X	·	00
stop	0	01
X	·	02
#	3	03
·	A	04
0	0	05
0	0	06
3	3	07
5	5	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

PERIOD RATE TO ANNUAL RATE

(settlement discount and credit cards)

Given:

Interest rate per period
Number of periods per year

Finds:

Equivalent annual rate

Execution:

number of periods per year / RUN / period rate / RUN / annual rate

e.g.

52 / RUN / weekly rate / RUN / annual rate
4 / RUN / quarterly rate / RUN / annual rate

Example:

A car dealer makes a credit agreement with a customer whereby £250 will be paid off in 30 fortnightly instalments of £10. He has used the program on page 54 to calculate that the effective fortnightly rate is 1·195%. Under the Consumer Credit Act, the equivalent annual rate must be specified. What is it?

Number of fortnights per year [2] [6] [RUN]
Fortnightly rate [1] [·] [1] [9] [5] [RUN]
Equivalent annual rate = 36·02%

X	·	00
(	6	01
stop	0	02
÷	G	03
#	3	04
1	1	05
0	0	06
0	0	07
+	E	08
#	3	09
1	1	10
=	—	11
ln	4	12
)	6	13
=	—	14
▼	A	15
e^x	4	16
—	F	17
#	3	18
1	1	19
X	·	20
#	3	21
1	1	22
0	0	23
0	0	24
=	—	25
stop	0	26
▼	A	27
goto	2	28
0	0	29
0	0	30
		31
		32
		33
		34
		35

Settlement discount

Example:

I can claim a discount of 2% if I settle an account due at the end of the month by the 15th of the month. What annual interest rate does this represent?

Solution:

Since months are of unequal lengths, take the period to be 1/2 month or 1/24 year.

Number of periods [2] [4] [RUN]
Period rate [2] [RUN]
Annual rate = 60·82% (rounded to nearest ·01%)

Credit Cards

Example:

I must pay 0·5% per week interest on my credit card account. What is the equivalent annual rate?

Number of periods [5] [2] [RUN]
Period rate [·] [5] [RUN]
Annual rate = 29·68% (rounded to nearest ·01%)

The same program may be used for calculating the period rate from the annual rate. Use the execution sequence:
number of periods / ÷ / RUN / annual rate / RUN / period rate

Example:

A bank charges 15% interest per annum. What is the equivalent quarterly rate?

Number of periods per year [4] [÷] [RUN]
Annual rate [1] [5] [RUN]
Result: Quarterly rate = 3·55% (rounded to nearest ·01%)

DAILY RATE TO ANNUAL RATE

X	·	00
#	3	01
3	3	02
·	A	03
6	6	04
5	5	05
=	—	06
▼	A	07
e^x	4	08
—	F	09
#	3	10
1	1	11
X	·	12
#	3	13
1	1	14
0	0	15
0	0	16
=	—	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Given:
Daily rate

Finds:
Annual rate

Execution:
daily rate / RUN / annual rate

Note: There is some loss of accuracy for daily rates of above about 0·3%.

ANNUAL RATE TO DAILY RATE

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
=	—	08
ln	4	09
÷	G	10
#	3	11
3	3	12
·	A	13
6	6	14
5	5	15
=	—	16
stop	0	17
▼	A	18
goto	2	19
0	0	20
0	0	21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Given:
Annual rate

Finds:
Daily rate

Execution:
annual rate / RUN / daily rate

Note: There is some loss of accuracy for annual rates of above about 200%.

MONTHLY RATE TO ANNUAL RATE

Given:
Monthly rate

Finds:
Equivalent annual rate

Comments:
Compounding every month

Execution:
monthly rate / RUN / annual rate

Example:
A dealer has calculated that the monthly interest rate on his H.P. agreements is 1·9%. Under the Consumer Credit Act he must display the annual rate. What is it?

[1] [·] [9] [RUN]

Result 25·32%

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
=	—	08
ln	4	09
X	·	10
#	3	11
1	1	12
2	2	13
=	—	14
▼	A	15
e^x	4	16
—	F	17
#	3	18
1	1	19
X	·	20
#	3	21
1	1	22
0	0	23
0	0	24
=	—	25
stop	0	26
▼	A	27
goto	2	28
0	0	29
0	0	30
		31
		32
		33
		34
		35

REGULAR REPAYMENT LOAN

Term of loan

Given:
Amount of loan
Amount of regular repayment
Interest rate

Finds:
Number of repayments

Comments:
Interest compounded every repayment period

Execution:
rate / RUN / amount of loan / RUN / repayment / RUN / number of repayments

Example:
I borrow £1000 at 10% interest. I repay £250 per year. How long will it take to pay off the debt?

Rate [1] [0] [RUN]
Initial sum [1] [0] [0] [0] [RUN]
Annual repayment [2] [5] [0] [RUN]
Answer 5·36 years

In practice I would make 5 payments of £250 and then pay off the balance outstanding; this can be worked out using the program on page 56.

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
X	·	05
sto	2	06
stop	0	07
÷	G	08
stop	0	09
—	F	10
#	3	11
1	1	12
—	F	13
÷	G	14
=	—	15
ln	4	16
÷	G	17
(	6	18
rcl	5	19
+	E	20
#	3	21
1	1	22
=	—	23
ln	4	24
)	6	25
=	—	26
stop	0	27
▼	A	28
goto	2	29
0	0	30
0	0	31
		32
		33
		34
		35

REGULAR REPAYMENT LOAN

Interest rate

Given:
Amount of loan
Amount of regular repayments
Number of repayments

Finds:
Interest rate per repayment period

Comments:
Interest compounded each repayment period

Formula:

$$I = \frac{100}{A_o}\left[1 - \frac{1}{\left(1 + \frac{I}{100}\right)^{N}}\right]$$

Execution:
repayment amount / RUN / amount of loan / RUN / number of repayments / RUN / estimate of rate / RUN / number of repayments / RUN / estimate of rate / RUN / number of repayments / RUN /

keep repeating until two successive values of the estimate of the interest rate are the same; this value is then the required interest rate.

X	·	00
#	3	01
1	1	02
0	0	03
0	0	04
÷	G	05
stop	0	06
÷	G	07
sto	2	08
#	3	09
1	1	10
0	0	11
0	0	12
+	E	13
#	3	14
1	1	15
=	—	16
ln	4	17
X	·	18
stop	0	19
—	F	20
=	—	21
▼	A	22
e^x	4	23
—	F	24
#	3	25
1	1	26
—	F	27
X	·	28
rcl	5	29
÷	G	30
stop	0	31
▼	A	32
goto	2	33
0	0	34
9	9	35

Example:
A television shop sells a £200 television on hire purchase terms of a £50 deposit followed by 18 monthly instalments of £10. Under the Consumer Credit Act, the shop is required to specify what interest rate this represents. What is the effective monthly interest rate?

Solution:
Amount of loan is £200 – £50 = £150

Repayment amount	[1] [0] [RUN]
Amount of loan	[1] [5] [0] [RUN]
Number of repayments	[1] [8] [RUN]
Estimate rate = 4.5805082	[RUN]
	[1] [8] [RUN]
Next estimate = 3.6899536	[RUN]

Repeat until two successive estimates are the same.
After several repetitions, reach the result of 1·9917271%.

Note: to obtain the equivalent annual rate, use the conversion program on page 52.

REGULAR REPAYMENT LOAN

Balance outstanding just after a repayment has been made

Given:
Amount of original loan
Amount of regular repayment
Number of repayments that have been made
Rate of interest per repayment period

Finds:
Amount outstanding

Comments:
Interest compounded each repayment period

Execution:
rate / RUN / number of repayments / RUN / repayment / RUN / original amount / RUN / balance

Example:
I borrowed £500 five years ago at 9% interest. I have repaid £100 each year since then. What will the balance be after this year's payment?

Rate	[9] [RUN]
Number of payments	[5] [RUN]
Payment	[1] [0] [0] [RUN]
Original amount	[5] [0] [0] [RUN]

So I now owe £170·83

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
sto	2	06
#	3	07
1	1	08
=	–	09
ln	4	10
X	·	11
stop	0	12
=	–	13
▼	A	14
e^x	4	15
X	·	16
(	6	17
stop	0	18
÷	G	19
rcl	5	20
=	–	21
sto	2	22
–	F	23
+	E	24
stop	0	25
)	6	26
+	E	27
rcl	5	28
=	–	29
stop	0	30
=	–	31
=	–	32
=	–	33
=	–	34
=	–	35

REGULAR REPAYMENT LOAN

Amount of repayment

Given:
Amount of loan
Number of repayment periods
Rate of interest

Finds:
Necessary regular repayment

Comments:
Interest compounded every repayment period

Execution:
rate / RUN / term / RUN / amount of loan / RUN / regular repayment

Example:
I take a loan of £100 at a rate of 1% per month. I want to pay back the money in 36 monthly instalments. How much do I pay per month?

Rate	[1] [RUN]
Term	[3] [6] [RUN]
Amount	[1] [0] [0] [RUN]

Regular repayment = £3·31
(rounded to nearest penny)

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
sto	2	06
#	3	07
1	1	08
=	–	09
ln	4	10
X	·	11
stop	0	12
=	–	13
▼	A	14
e^x	4	15
÷	G	16
–	F	17
#	3	18
1	1	19
–	F	20
÷	G	21
rcl	5	22
÷	G	23
stop	0	24
÷	G	25
=	–	26
stop	0	27
▼	A	28
goto	2	29
0	0	30
0	0	31
		32
		33
		34
		35

SINGLE REPAYMENT LOAN

Final amount

Given:
Rate of interest per accounting period
Number of accounting periods
Initial sum

To find:
Final sum

Comments:
Interest compounded each accounting period

Execution:
rate of interest / RUN / number of periods / RUN / initial sum / RUN / final sum

Formula:

$$F = I\left(1 + \frac{\alpha}{100}\right)^n$$

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
=	–	08
ln	4	09
X	·	10
stop	0	11
=	–	12
▼	A	13
e^x	4	14
X	·	15
stop	0	16
=	–	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
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SINGLE REPAYMENT LOAN

Final amount

Given:
Annual rate of interest
Term of loan
Initial sum

To find:
Final sum

Comments:
Interest compounded every six months

Execution:
rate of interest / RUN / term in years / RUN / initial sum / RUN / final sum

Example:
I invest £570 at 8% interest. How much is in my account after 5 years?

Rate of interest	[8] [RUN]
Term in years	[5] [RUN]
Initial sum	[5] [7] [0] [RUN]

Answer £843·65

Formula:

$$F = I\left(1 + \frac{a}{100}\right)^{2n}$$

÷	G	00
#	3	01
2	2	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
=	–	08
ln	4	09
X	·	10
stop	0	11
+	E	12
=	–	13
▼	A	14
e^x	4	15
X	·	16
stop	0	17
=	–	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
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		35

SINGLE REPAYMENT LOAN

Number of years to achieve given result

Given:
Initial sum
Final sum
Rate of interest per accounting period

Finds:
Number of accounting periods

Comments:
Interest compounded each accounting period

Execution:
rate / RUN / initial sum / RUN / final sum / RUN / term

Example:
How long will it take £700 to become £2000 if interest of 12½% is paid annually?

Rate [1] [2] [·] [5] [RUN]
Initial sum [7] [0] [0] [RUN]
Final sum [2] [0] [0] [0] [RUN]

Answer 8·916 years; so the first time the balance will exceed £2000 will be after the ninth interest payment.

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
=	–	08
ln	4	09
sto	2	10
stop	0	11
÷	G	12
stop	0	13
÷	G	14
=	–	15
ln	4	16
÷	G	17
rcl	5	18
=	–	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
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SINGLE REPAYMENT LOAN

Number of years to achieve given result

Given:
Initial sum
Final sum
Annual rate of interest

Finds:
Term

Comments:
Interest compounded every six months

Execution:
rate / RUN / initial sum / RUN / final sum / RUN / term

÷	G	00
#	3	01
2	2	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
=	–	08
ln	4	09
sto	2	10
stop	0	11
÷	G	12
stop	0	13
÷	G	14
=	–	15
ln	4	16
÷	G	17
rcl	5	18
÷	G	19
#	3	20
2	2	21
=	–	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
0	0	27
		28
		29
		30
		31
		32
		33
		34
		35

SINGLE REPAYMENT LOAN

Interest rate needed for given result

Given:
Number of accounting periods
Initial and final sum

Finds:
Effective rate of interest per accounting period

Comments:
Interest compounded every accounting period

Execution:
initial sum / RUN / final sum / RUN / term / RUN / rate of interest

÷	G	00
stop	0	01
÷	G	02
=	–	03
ln	4	04
÷	G	05
stop	0	06
=	–	07
▼	A	08
e^x	4	09
–	F	10
#	3	11
1	1	12
X	·	13
#	3	14
1	1	15
0	0	16
0	0	17
=	–	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
		25
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		27
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SINGLE REPAYMENT LOAN

Interest rate for given result

Given:
Term in years
Initial and final sum

Finds:
Effective annual interest rate

Comments:
Interest compounded every six months

Execution:
initial sum / RUN / final sum / RUN / term / RUN / rate of interest

Example:
A bond costs £100 and is repayable in 4 years at £150. What rate of interest does this represent?

Initial sum [1] [0] [0] [RUN]
Final sum [1] [5] [0] [RUN]
Term [4] [RUN]
Equivalent interest rate = 10·38%

÷	G	00
stop	0	01
÷	G	02
=	–	03
ln	4	04
÷	G	05
(	6	06
stop	0	07
+	E	08
)	6	09
=	–	10
▼	A	11
e^x	4	12
–	F	13
#	3	14
1	1	15
X	·	16
#	3	17
2	2	18
0	0	19
0	0	20
=	–	21
stop	0	22
▼	A	23
goto	2	24
0	0	25
0	0	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

PRESENT VALUE OF A SINGLE FUTURE PAYMENT

Given:

Rate of interest per accounting period

Number of periods ahead that payment is to be made

Finds:

Present value of future payment

Comments:

Interest compounded every accounting period

Execution:

rate / RUN / term / RUN / amount / RUN /
present value

Formula:

$$I = \frac{F}{\left(1 + \frac{a}{100}\right)^{n}}$$

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
÷	G	08
=	—	09
ln	4	10
X	·	11
stop	0	12
=	—	13
▼	A	14
e^x	4	15
X	·	16
stop	0	17
=	—	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

PRESENT VALUE OF A SINGLE FUTURE PAYMENT

Given:

Annual rate of interest

Number of years ahead that payment is to be made

Amount of payment

Finds:

Present value

Comments:

Interest compounded every six months

Execution:

rate / RUN / term / RUN / amount / RUN /
present value

Example:

What is the present value of a payment of £5000 made in 4 years time at an annual rate of 14%?

Rate [1] [4] [RUN]

Term [4] [RUN]

Amount [5] [0] [0] [0] [RUN]

Answer: present value = £2909·67

Formula:

$$I = \frac{F}{\left(1 + \frac{a}{200}\right)^{2n}}$$

÷	G	00
#	3	01
2	2	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
÷	G	08
=	—	09
ln	4	10
X	·	11
sto	2	12
stop	0	13
+	E	14
=	—	15
▼	A	16
e^x	4	17
X	·	18
stop	0	19
=	—	20
stop	0	21
▼	A	22
goto	2	23
0	0	24
0	0	25
		26
		27
		28
		29
		30
		31
		32
		33
		34
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PRESENT VALUE OF A SERIES OF POSSIBLE UNEQUAL FUTURE PAYMENTS

Given:

Payments
Interest rate per payment period

Finds:

Present value

Execution:

Suppose payments are made of p_1 at the end of the first year, p_2 at the end of the second year, and so on up to a final payment of p_n at the end of the nth year.

Use the following execution sequence:
interest rate / RUN / p_n / RUN / · · · / RUN / p_1 / RUN / present value of all future payments

Before a new calculation:
C/CE / ▲▼ / ▲▼ / goto / 0 / 0 /

Notice that the payments are entered *in reverse order,* with the last payment first.

Example:

An investor wishes to make future payments to a businessman as follows:

1 Jan.	1978	£10,000
	1979	£12,000
	1980	£15,000
	1981	£20,000
	1982	£20,000

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
#	3	06
1	1	07
÷	G	08
=	–	09
sto	2	10
stop	0	11
X	·	12
rcl	5	13
+	E	14
▼	A	15
goto	2	16
1	1	17
1	1	18
		19
		20
		21
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Reckoning the annual interest rate to be 14%, what is the value of these payments on 1 Jan. 1977?

Rate	[1] [4] [RUN]
1982	[2] [0] [0] [0] [0] [RUN]
1981	[2] [0] [0] [0] [0] [RUN]
1980	[1] [5] [0] [0] [0] [RUN]
1979	[1] [2] [0] [0] [0] [RUN]
1978	[1] [0] [0] [0] [0] [RUN]

Payments in reverse order

Present value = £50,359

PRESENT VALUE OF A SERIES OF EQUAL FUTURE PAYMENTS

Given:

Rate of interest per payment period
Number of payments
Amount of each payment

Finds:

Present value

Comments:

Assumes payments start at the end of the first payment period
Interest compounded each payment period

Execution:

rate / RUN / number of payments / RUN / amount of each payment / RUN / present value

Example:

Find the present value of £200,000 paid in 20 equal annual instalments. The rate of interest is 13% and the first payment is made immediately.

Solution:

There are 19 equal future payments of £10,000 and one present payment. Find the present value of the future payments first and then add the present payment.

Rate	[1] [3] [RUN]
Number of payments	[1] [9] [RUN]
Amount	[1] [0] [0] [0] [0] [RUN]
Add present payment	[+] [1] [0] [0] [0] [0] [=]

So present value of all payments is £79,379

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
sto	2	06
#	3	07
1	1	08
=	–	09
ln	4	10
X	·	11
stop	0	12
–	F	13
=	–	14
▼	A	15
e^x	4	16
–	F	17
#	3	18
1	1	19
–	F	20
÷	G	21
rcl	5	22
X	·	23
stop	0	24
=	–	25
stop	0	26
▼	A	27
goto	2	28
0	0	29
0	0	30
		31
		32
		33
		34
		35

PRESENT VALUE OF A SERIES OF EQUAL PAYMENTS FOLLOWED BY A SINGLE PAYMENT

(e.g. Dated government stocks)

Given:

Regular payment (paid at the end of each repayment period including the last)
Final payment (excluding final regular payment)
Number of repayment periods
Discounting interest rate per repayment period

Finds:

Present value of future payments

Comments:

Notional interest compounded each repayment period.

Execution:

interest rate / RUN / no. of repayments / RUN / regular payment / RUN / final payment / RUN / present value

Example:

What is the present value of a government stock which yields £35 every half year and will be repaid at £1000 in 8½ years time? Take interest rate for discounting to be 6½% per half year.

Rate	[6] [·] [5] [RUN]
Number of repayments	[1] [9] [RUN]
Regular payment	[3] [5] [RUN]
Final payment	[1] [0] [0] [0] [RUN]

Present value = £677·96

÷	G	00
#	3	01
1	1	02
0	0	03
0	0	04
+	E	05
sto	2	06
#	3	07
1	1	08
=	–	09
ln	4	10
X	·	11
stop	0	12
–	F	13
=	–	14
▼	A	15
e^x	4	16
▼	A	17
MEx	5	18
÷	G	19
X	·	20
stop	0	21
=	–	22
▼	A	23
MEx	5	24
X	·	25
(	6	26
stop	0	27
–	F	28
rcl	5	29
)	6	30
+	E	31
rcl	5	32
=	–	33
stop	0	34
=	–	35

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MEAN AND STANDARD DEVIATION

Observations $x_1, \cdots, x_n$

Mean $\bar{x} = \frac{1}{n}\Sigma x_i$

(i) Standard deviation about mean

$$\sigma = \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}$$

(ii) Standard deviation about a

$$\sigma_a = \sqrt{\frac{1}{n}\Sigma(x_i - a)^2}$$

Execution:

(i) RUN / x_1 / RUN / x_2 / · · · / x_n / RUN / ▲▼ / ▲▼ / goto / 1 / 9 / RUN * / n / RUN / $\bar{x}$ / RUN / σ

(ii) as (i) to *, then · · · / n / RUN / $\bar{x}$ / a / RUN / σ_a

#	3	00
0	0	01
=	—	02
sto	2	03
(	6	04
stop	0	05
+	E	06
▼	A	07
MEx	5	08
=	—	09
▼	A	10
MEx	5	11
X	·	12
)	6	13
+	E	14
▼	A	15
goto	2	16
0	0	17
4	4	18
rcl	5	19
÷	G	20
stop	0	21
sto	2	22
=	—	23
stop	0	24
X	·	25
X	·	26
rcl	5	27
—	F	28
)	6	29
÷	G	30
rcl	5	31
=	—	32
$\sqrt{x}$	1	33
stop	0	34
=	—	35

MEAN, SUM OF SQUARES ABOUT MEAN, AND ESTIMATE OF VARIANCE

Mean $\bar{x} = \frac{1}{n}\Sigma x_i$

Sum of squares about mean $S_{xx} = \Sigma(x_i - \bar{x})^2$

Estimate of variance $s^2 = \frac{S_{xx}}{n-1}$

Pre-execution:

Before each set of data is entered, clear memory with / C/CE / ▲▼ / sto /

Execution:

RUN / x_1 / RUN / x_2 / · · · / x_n / RUN / Σx^2 / ▲▼ / ▲▼ / goto / 1 / 5 / RUN / Σx / n / RUN / $\bar{x}$ / RUN / S_{xx} / RUN / s^2

(	6	00
stop	0	01
+	E	02
▼	A	03
MEx	5	04
=	—	05
▼	A	06
MEx	5	07
X	·	08
)	6	09
+	E	10
▼	A	11
gotc	2	12
0	0	13
0	0	14
rcl	5	15
÷	G	16
stop	0	17
sto	2	18
X	·	19
stop	0	20
X	·	21
rcl	5	22
—	F	23
)	6	24
÷	G	25
stop	0	26
(	6	27
rcl	5	28
—	F	29
#	3	30
1	1	31
=	—	32
)	6	33
=	—	34
stop	0	35

LINEAR REGRESSION AND CORRELATION COEFFICIENT

Observations $(x_1, y_1), \cdots, (x_n, y_n)$

Sum of cross products $S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y})$

Correlation coefficient

$$r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2 \Sigma(y_i - \bar{y})^2}}$$

Regression line (y on x) $y = a + bx$

Method:

First use program on page 73 applied to the x's and y's separately to calculate $\bar{x}$, S_{xx}, $\bar{y}$ and S_{yy}. Then use this program as follows.

Execution:

$\bar{x}$ / RUN / x_1 / RUN / y_1 / RUN / x_2 / RUN / y_2 / $\cdots$ / x_n / RUN / y_n / ▲▼ /) / = / ▲▼ / ▲▼ / goto / 1 / 3 / RUN / S_{xy} / S_{xx} / RUN / S_{yy} / RUN / r / RUN / b / $\bar{x}$ / RUN / $\bar{y}$ / RUN / a

sto	2	00
(	6	01
stop	0	02
—	F	03
rcl	5	04
X	·	05
stop	0	06
)	6	07
+	E	08
▼	A	09
goto	2	10
0	0	11
1	1	12
÷	G	13
(	6	14
stop	0	15
÷	G	16
stop	0	17
sto	2	18
=	—	19
√x	1	20
X	·	21
▼	A	22
MEx	5	23
=	—	24
)	6	25
÷	G	26
stop	0	27
rcl	5	28
X	·	29
stop	0	30
—	F	31
stop	0	32
—	F	33
=	—	34
stop	0	35

SLOPE OF REGRESSION LINE

Regression line is $y = a + bx$

Observations $(x_1, y_1), (x_2, y_2), \cdots (x_n, y_n)$

Execution:

RUN / y_1 / RUN / x_1 / RUN / y_2 / RUN / x_2 / RUN / $\cdots$ / x_n / RUN / Σxy (note) / C/CE / ▲▼ / ▲▼ / MEx / Σy (note) / ▲▼ / (/ x_1 / RUN / RUN / x_2 / RUN / RUN / $\cdots$ / x_n / RUN / ▲▼ /) / + / ▲▼ / ▲▼ / goto / 1 / 6 / RUN / n / RUN / Σy / RUN / Σxy / RUN / b

Note: The values of Σxy and Σy must be written down and re-entered later in the execution sequence.

(	6	00
stop	0	01
+	E	02
▼	A	03
MEx	5	04
=	—	05
▼	A	06
MEx	5	07
X	·	08
stop	0	09
)	6	10
+	E	11
▼	A	12
goto	2	13
0	0	14
0	0	15
(	6	16
rcl	5	17
—	F	18
÷	G	19
stop	0	20
X	·	21
▼	A	22
MEx	5	23
)	6	24
÷	G	25
X	·	26
(	6	27
rcl	5	28
X	·	29
stop	0	30
+	E	31
stop	0	32
)	6	33
=	—	34
stop	0	35

TESTING THE HYPOTHESIS OF ZERO CORRELATION

÷	G	00
(	6	01
X	·	02
—	F	03
#	3	04
1	1	05
—	F	06
=	—	07
$\sqrt{x}$	1	08
)	6	09
X	·	10
(	6	11
stop	0	12
—	F	13
#	3	14
2	2	15
=	—	16
$\sqrt{x}$	1	17
)	6	18
=	—	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Assuming normality, on the hypothesis that $\rho = 0$, the statistic

$$t = r\,\frac{\sqrt{N-2}}{\sqrt{1-r^2}}$$

has the t distribution with (N — 2) degrees of freedom. Large values of t indicate that the true correlation coefficient is non-zero.

Execution:

r / RUN / N / RUN / t

REGRESSION LINE SLOPE

To test whether it is significantly different from zero or any other given value b_0

—	F	00
stop	0	01
—	F	02
÷	G	03
(	6	04
stop	0	05
X	·	06
—	F	07
#	3	08
1	1	09
—	F	10
=	—	11
$\sqrt{x}$	1	12
)	6	13
X	·	14
(	6	15
stop	0	16
—	F	17
#	3	18
2	2	19
=	—	20
$\sqrt{x}$	1	21
)	6	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

Slope of regression line = b

Correlation coefficient = r

Sample size = N

Calculate the statistic

$$t = \frac{(b - b_0)\sqrt{N-2}}{\sqrt{1-r^2}}$$

On the null hypothesis that the true value of b is b_0, this has the t-distribution with (N — 2) degrees of freedom (approximately standard normal if N is reasonably large).

Execution:

b_0 / RUN / b / RUN / r / RUN / N / RUN / t

If b_0 is zero the following can be used:

b / RUN / RUN / r / RUN / N / RUN / t

STUDENT'S t–TEST

$$t = \frac{\bar{x}\sqrt{n}}{s}$$

To test whether the mean of a set of observations $x_1, \cdots, x_n$ differs significantly from zero. Large values of t reject the hypothesis that the mean is zero.

Pre-execution:

Clear memory with C/CE / ▲▼ / sto /

Execution:

RUN / x_1 / RUN / x_2 / · · · / x_n / RUN / ▲▼ / ▲▼ / goto / 1 / 5 / RUN / n / RUN / n / RUN / t

To re-use:

C/CE / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

(	6	00
stop	0	01
+	E	02
▼	A	03
MEx	5	04
=	—	05
▼	A	06
MEx	5	07
X	·	08
)	6	09
+	E	10
▼	A	11
goto	2	12
0	0	13
0	0	14
rcl	5	15
X	·	16
÷	G	17
stop	0	18
—	F	19
)	6	20
÷	G	21
(	6	22
stop	0	23
÷	G	24
—	F	25
#	3	26
1	1	27
—	F	28
)	6	29
=	—	30
$\sqrt{x}$	1	31
÷	G	32
X	·	33
rcl	5	34
=	—	35

STUDENT'S t–TEST

To test whether the mean is significantly different from some value a:

$$t = \frac{(\bar{x} - a)\sqrt{n}}{s}$$

Pre-execution (before each set of data):

/ ▲▼ / ▲▼ / goto / 0 / 0 / C/CE / C/CE / ▲▼ / sto /

Execution:

RUN / x_1 / RUN / x_2 / · · · / x_n / RUN / ▲▼ / ▲▼ / goto / 1 / 5 / RUN / n / RUN / n / RUN / n – 1 / RUN / a / RUN / = / t

(	6	00
stop	0	01
+	E	02
▼	A	03
MEx	5	04
=	—	05
▼	A	06
MEx	5	07
X	·	08
)	6	09
+	E	10
▼	A	11
goto	2	12
0	0	13
0	0	14
rcl	5	15
÷	G	16
stop	0	17
X	·	18
▼	A	19
MEx	5	20
—	F	21
)	6	22
÷	G	23
X	·	24
stop	0	25
X	·	26
stop	0	27
=	—	28
$\sqrt{x}$	1	29
X	·	30
(	6	31
rcl	5	32
—	F	33
stop	0	34
=	—	35

CHI–SQUARED

Observed values $O_1, \cdots, O_n$

Expected values $E_1, \cdots, E_n$

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i}$$

Execution:

RUN / O_1 / RUN / E_1 / RUN / O_2 / RUN / E_2 / $\cdots$ / O_n / RUN / E_n / RUN / χ^2

For new data:

Clear with C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0 /

(	6	00
stop	0	01
–	F	02
stop	0	03
sto	2	04
×	·	05
÷	G	06
rcl	5	07
)	6	08
+	E	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CHI–SQUARED WITH YATES CORRECTION

(e.g. for small contingency tables)

$$\chi^2 = \Sigma \frac{(|O_i - E_i| - \frac{1}{2})^2}{E_i}$$

Execution:

RUN / O_1 / RUN / E_1 / RUN / O_2 / RUN / E_2 / $\cdots$ / O_n / RUN / E_n / RUN / χ^2

(	6	00
stop	0	01
–	F	02
stop	0	03
sto	2	04
×	·	05
=	–	06
√x	1	07
–	F	08
#	3	09
·	A	10
5	5	11
×	·	12
÷	·	13
rcl	5	14
)	6	15
+	E	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

TWO SAMPLE CHI–SQUARED

+	E	00
stop	0	01
÷	G	02
X	·	03
(	6	04
+	E	05
÷	G	06
—	F	07
stop	0	08
+	E	09
X	·	10
)	6	11
+	E	12
rcl	5	13
=	—	14
sto	2	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

$$\chi^2 = \Sigma \frac{(O_i - O_i')^2}{O_i + O_i'}$$

Pre-execution:

Clear memory with C/CE / ▲▼ / sto /

Execution:

O_1 / RUN / O_1' / RUN / O_1 / RUN / O_2 / RUN / O_2' / RUN / O_2 / · · · / O_n / RUN / O_n' / RUN / O_n / RUN / χ^2

TWO SAMPLE CHI–SQUARED WITH YATES CORRECTION

+	E	00
stop	0	01
÷	G	02
X	·	03
(	6	04
+	E	05
÷	G	06
—	F	07
stop	0	08
+	E	09
X	·	10
=	—	11
√x	1	12
—	F	13
#	3	14
1	1	15
X	·	16
)	6	17
+	E	18
rcl	5	19
=	—	20
sto	2	21
stop	0	22
▼	A	23
goto	2	24
0	0	25
0	0	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

$$\chi^2 = \Sigma \frac{(|O_i - O_i'| - 1)^2}{O_i + O_i'}$$

Execution:

O_1 / RUN / O_1' / RUN / O_1 / RUN / O_2 / RUN / O_2' / RUN / O_2 / · · · / O_n / RUN / O_n' / RUN / O_n / RUN / χ^2

Caution:

If for any j, $O_j = O_j' = 0$, do not enter either of them but go straight on to O_{j+1}. In any case it is not very sound statistically to use the χ^2 if any of the $(O_j + O_j')$ are less than about 10.

CONTINGENCY TABLE: χ^2 –TEST FOR INDEPENDENCE

Given a contingency table with h rows and k columns, and observation O_{ij} at the intersection of the ith row and the jth column, it is often of interest to test whether the 'row effect' and 'column effect' are independent. To do this, proceed as follows:

1. Work out the row totals R_i, the column totals C_j and the grand total N.
2. Use the program opposite to calculate the expected values E_{ij} for each cell in the table.
3. Use one of the one-sample χ^2 programs above to work out the χ^2 statistic defined by

$$\Sigma \frac{(O - E)^2}{E} \qquad \text{or} \qquad \Sigma \frac{(|O - E| - \frac{1}{2})^2}{E}$$

Make sure that the observed and expected values are entered for every cell of the table. Use the Yates corrected version if the table is small. The number of degrees of freedom is (h – 1)(k – 1). If this is fairly large the resulting statistic may be transformed to have a standard normal distribution on the hypothesis of independence by using the transformation program on page 93.

CALCULATING THE EXPECTED VALUES IN A CONTINGENCY TABLE

$$E_{ij} = \frac{R_i C_j}{n}$$

Execution:

N / RUN / R_1 / RUN / C_1 / RUN / E_{11} / RUN / RUN / C_2 / RUN / E_{12} / · · · / E_{1k} / RUN / R_2 / RUN / C_1 / RUN / E_{21} / · · · etc.

The current row total is displayed between the two successive / RUN / steps after each result is displayed. It should be altered at this point when moving on from one row to the next.

sto	2	00
stop	0	01
+	E	02
(	6	03
X	·	04
stop	0	05
÷	G	06
rcl	5	07
=	–	08
stop	0	09
#	3	10
0	0	11
=	–	12
)	6	13
=	–	14
▼	A	15
goto	2	16
0	0	17
1	1	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Z STATISTIC

For testing whether a proportion is significantly different from θ. The statistic Z has mean 0 and variance 1 and is approximately normally distributed.

$$Z = \frac{\frac{x}{n} - \theta}{\sqrt{\frac{\theta(1-\theta)}{n}}}$$

Execution:

θ / RUN / n / RUN / x / RUN / z

sto	2	00
—	F	01
(	6	02
X	·	03
)	6	04
=	—	05
$\sqrt{x}$	1	06
÷	G	07
X	·	08
(	6	09
rcl	5	10
X	·	11
stop	0	12
sto	2	13
—	F	14
stop	0	15
—	F	16
)	6	17
÷	G	18
(	6	19
rcl	5	20
$\sqrt{x}$	1	21
)	6	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

NON-PARAMETRIC STATISTICS

Spearman's rank correlation coefficient

Pairs of ranks
$(r_1, s_1), (r_2, s_2), \cdots, (r_n, s_n)$

Execution:

n / RUN / r_1 / RUN / s_1 / RUN / · · · / r_n / RUN / s_n / RUN / ρ

sto	2	00
X	·	01
X	·	02
rcl	5	03
—	F	04
rcl	5	05
—	F	06
÷	G	07
#	3	08
6	6	09
=	—	10
sto	2	11
#	3	12
1	1	13
+	E	14
(	6	15
stop	0	16
—	F	17
stop	0	18
X	·	19
÷	G	20
rcl	5	21
)	6	22
▼	A	23
goto	2	24
1	1	25
4	4	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

QUALITY CONTROL

Action and warning limits for proportion of batch having given attribute.

$$a\pm = p \pm \alpha\sqrt{\frac{p(1-p)}{n}}$$

Typical values of α:

For action limits $\alpha = 3{\cdot}12$

For warning limits $\alpha = 1{\cdot}96$

Execution:

p / RUN / n / RUN / α / RUN / a− / RUN / a+

sto	2	00
—	F	01
(	6	02
X	·	03
)	6	04
÷	G	05
stop	0	06
=	—	07
$\sqrt{x}$	1	08
X	·	09
stop	0	10
=	—	11
▼	A	12
MEx	5	13
—	F	14
rcl	5	15
+	E	16
stop	0	17
rcl	5	18
+	E	19
rcl	5	20
=	—	21
stop	0	22
▼	A	23
goto	2	24
0	0	25
0	0	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

NORMAL DENSITY FUNCTION

$$\phi = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

Execution:

x / RUN / μ / RUN / σ / RUN / ϕ

—	F	00
stop	0	01
÷	G	02
stop	0	03
sto	2	04
X	·	05
—	F	06
=	—	07
▼	A	08
e^x	4	09
÷	G	10
#	3	11
6	6	12
·	A	13
2	2	14
8	8	15
3	3	16
1	1	17
9	9	18
=	—	19
$\sqrt{x}$	1	20
÷	G	21
rcl	5	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

Given any α with $0 < \alpha < 0{\cdot}5$, finds x to within about 2 sig. fig. so that the probability that a standard normal random variable exceeds x is α.

Execution:

α / RUN / x

For greater accuracy (·1% error) divide result by 1·006.

For still greater accuracy use execution sequence

α / X / 1·0007 / RUN / ÷ / 1·006 / = / x

X	·	00
÷	G	01
=	—	02
ln	4	03
$\sqrt{x}$	1	04
sto	2	05
+	E	06
+	E	07
+	E	08
#	3	09
1	1	10
2	2	11
·	A	12
5	5	13
÷	G	14
(	6	15
rcl	5	16
+	E	17
#	3	18
7	7	19
X	·	20
rcl	5	21
+	E	22
#	3	23
5	5	24
=	—	25
)	6	26
—	F	27
+	E	28
rcl	5	29
=	—	30
stop	0	31
▼	A	32
goto	2	33
0	0	34
0	0	35

POISSON DISTRIBUTION

Suppose a random variable has the Poisson distribution with parameter λ. What is the probability that the random variable takes the value j?

Formula:

$$\text{prob (j)} = \frac{e^{-\lambda}\lambda^{j}}{j!}$$

Execution:

λ / RUN / j / RUN / answer·

Note: Long execution times are possible for large values of j.

—	F	00
(	6	01
ln	4	02
X	·	03
stop	0	04
sto	2	05
)	6	06
—	F	07
—	F	08
(	6	09
rcl	5	10
—	F	11
#	3	12
1	1	13
+	E	14
▼	A	15
gin	1	16
2	2	17
9	9	18
sto	2	19
#	3	20
1	1	21
=	—	22
ln	4	23
)	6	24
▼	A	25
goto	2	26
0	0	27
8	8	28
=	—	29
rcl	5	30
)	6	31
=	—	32
▼	A	33
e^x	4	34
stop	0	35

FISHER'S Z TRANSFORMATION FOR CORRELATION COEFFICIENTS.

$$z = \tfrac{1}{2}\log\left(\frac{1+\rho}{1-\rho}\right)$$

The distribution of z is approximately normal.

Execution:

ρ / RUN / z / n / RUN / σ

where n is the sample size and σ is the standard deviation of z.

$$\sigma = \frac{1}{\sqrt{n-3}}$$

—	F	00
#	3	01
1	1	02
÷	G	03
+	E	04
+	E	05
#	3	06
1	1	07
—	F	08
=	—	09
$\sqrt{x}$	1	10
ln	4	11
stop	0	12
—	F	13
#	3	14
3	3	15
÷	G	16
=	—	17
$\sqrt{x}$	1	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

TRANSFORMING χ^2 TO NORMAL

Suppose x has the χ^2 distribution with n degrees of freedom, where n is fairly large (say $n \geqslant 20$).

Then $y = \sqrt{2x^2} - \sqrt{2n-1}$ has approximately a standard normal distribution with mean 0 and variance 1.

Execution:

x / RUN / n / RUN / y

×	·	00
+	E	01
=	—	02
$\sqrt{x}$	1	03
—	F	04
(	6	05
stop	0	06
+	E	07
—	F	08
#	3	09
1	1	10
=	—	11
$\sqrt{x}$	1	12
)	6	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
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TRANSFORMING BINOMIAL TO NORMAL

Suppose x is binomially distributed with parameters n and p. Then

$$z = \frac{\frac{x}{n} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

has very nearly a standard normal distribution provided np and n(1 – p) are both greater than 5.

Execution:

p / RUN / n / RUN / x / RUN / z

sto	2	00
–	F	01
(	6	02
×	·	03
)	6	04
=	–	05
√x	1	06
÷	G	07
×	·	08
(	6	09
rcl	5	10
×	·	11
stop	0	12
sto	2	13
–	F	14
stop	0	15
–	F	16
)	6	17
÷	G	18
(	6	19
rcl	5	20
√x	1	21
)	6	22
=	–	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

		00
		01
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2 Mathematics

Program Library

Algebra
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Geometry
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Mathematics

2

CONTENTS

How to use these programs

Each program is arranged as follows:

1. On the left of the page, explanatory information and the 'execution sequence', the sequence of keystrokes necessary for running the program. Results displayed are printed in gold.
2. In the first column on the right hand side of the page, the sequence of keystrokes which make up the program.
3. In the second and third columns on the right hand side of the page, the program in check symbol and step number form (see section on checking the program).

Notes

1. Where a key has more than one function, the relevant function is printed as the keystroke in the first column

   e.g. the keystroke [8] (cos / arccos) may appear as 8, cos or arccos.
2. The symbol ▼ within a program always refers to the key [·/EE/–] (▼)
3. The symbol # refers to [3] (ChN/#)
4. The abbreviation gin is 'go if neg' and so refers to the key [1] (go if neg)

Entering the program

To enter a program into the calculator:

1. Press [▲▼] [▲▼] [2] (go to) [0] [0]	Display shows step programmed at 00 in check symbol form as described below.
2. Press [▲▼] [RUN] (learn)	No change in display.
3. Press the sequence of keys for the program as shown in the first column of the program page.	At each stage the step about to be overwritten is displayed. When the machine is first switched on every step is zero.
4. Press [C/CE]	Normal number display is resumed.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]	The step programmed at 00 will be displayed.

Checking the program

Each of the programs in the library is shown in check symbol form in the second column on the right-hand side of the page.

Press [▲▼] [C/CE] (step) repeatedly, and at each stage the check symbol will appear on the left of the display with the step number on the right. Ignore the four zeros in the display.

e.g. A.0000 03 (A = check symbol; 03 = step number)

After stepping through the program, press

[▲▼] [▲▼] [2] (go to) [0] [0] before execution.

Finally, press [C/CE] and the program is ready for use.

Correcting the program

If the check symbol for a particular step number is not as indicated in the last two columns of the program page:

1. Press [▲▼] [▲▼] [2] (go to) followed by the step number if the appropriate step number is not already displayed.
2. Press [▲▼] [RUN] (learn)
3. Enter the correct keystroke. The display will then show the next step in the program. If this is also incorrect, enter the correct keystroke. At each stage, the step about to be overwritten will be displayed.
4. When correction has been completed, press [C/CE]. Any step which has not been overwritten will not be affected.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]

Note
To restore normal use of the calculator after entering or checking the program, press [C/CE]

Running the program

Press the sequence of keys as shown in the program library in the execution sequence. Results displayed are printed in gold.

EXTENSION OF RANGE OF TRIGONOMETRIC FUNCTIONS

to $-\pi < \theta < \pi$

Sine of any angle:

$$\sin\theta = \frac{2t}{1+t^2} \quad \text{where } t = \tan\frac{\theta}{2}$$

Execution:

θ / RUN / sin θ

For θ in degrees, insert / ▼ / D→R / at start of program.

÷	G	00
#	3	01
2	2	02
=	—	03
tan	9	04
÷	G	05
(	6	06
X	·	07
+	E	08
#	3	09
1	1	10
=	—	11
)	6	12
+	E	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
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EXTENSION OF RANGE OF TRIGONOMETRIC FUNCTIONS

to $-\pi < \theta < \pi$

Cosine of any angle

$$\cos\theta = \frac{1-t^2}{1+t^2} \quad \text{where } t = \tan\frac{\theta}{2}$$

Execution:

θ / RUN / cos θ

÷	G	00
#	3	01
2	2	02
=	—	03
tan	9	04
X	·	05
+	E	06
#	3	07
1	1	08
÷	G	09
+	E	10
−	F	11
#	3	12
1	1	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
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EXTENSION OF RANGE OF TRIGONOMETRIC FUNCTIONS

to $-\pi < \theta < \pi$

Tangent of any angle

$$\tan\theta = \frac{2t}{1-t^2} \quad \text{where } t = \tan\frac{\theta}{2}$$

Execution:

θ / RUN / $\tan\theta$

÷	G	00
#	3	01
2	2	02
=	–	03
tan	9	04
÷	G	05
(	6	06
X	·	07
–	F	08
#	3	09
1	1	10
–	F	11
)	6	12
+	E	13
=	–	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
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EXTENSION OF RANGE OF TRIGONOMETRIC FUNCTIONS

to $-\pi < \theta < \pi$

sin, cos and tan using $t = \tan\frac{\theta}{2}$

Execution:

θ / RUN / $\sin\theta$ / RUN / $\cos\theta$ / RUN / $\tan\theta$

÷	G	00
#	3	01
2	2	02
=	–	03
tan	9	04
sto	2	05
X	·	06
+	E	07
#	3	08
1	1	09
÷	G	10
=	–	11
▼	A	12
MEx	5	13
X	·	14
rcl	5	15
+	E	16
=	–	17
stop	0	18
▼	A	19
MEx	5	20
+	E	21
–	F	22
#	3	23
1	1	24
÷	G	25
stop	0	26
X	·	27
rcl	5	28
=	–	29
stop	0	30
▼	A	31
goto	2	32
0	0	33
0	0	34
		35

SINE AND COSINE OF ANY ANGLE

Sin: use program on right

Execution:

angle in degrees / RUN / sine

For radians version of program, insert / ▼ / R→D / at beginning and omit / = / = / at end.

Cos: *either* use program on right and execute by / ▲▼ / ▲▼ / goto / 0 / 4 / angle in degrees / RUN / cosine

or omit first four keystrokes of program on right and fill the empty spaces at the end with repeated / = / and execute by

angle in degrees / RUN / cosine

For radians version of program, insert / ▼ / R→D / at the beginning.

Note: E can appear if reduced angles > 1·57 radians.

—	F	00
#	3	01
9	9	02
0	0	03
X	·	04
=	—	05
$\sqrt{x}$	1	06
—	F	07
+	E	08
#	3	09
3	3	10
6	6	11
0	0	12
—	F	13
▼	A	14
gin	1	15
0	0	16
7	7	17
#	3	18
1	1	19
8	8	20
0	0	21
X	·	22
=	—	23
$\sqrt{x}$	1	24
—	F	25
#	3	26
9	9	27
0	0	28
=	—	29
▼	A	30
D→R	3	31
sin	7	32
stop	0	33
=	—	34
=	—	35

TANGENT OF ANY ANGLE

Execution:

angle in degrees / RUN / tangent

Note: E can appear if reduced angle > 1·57 radians.

+	E	00
#	3	01
9	9	02
0	0	03
÷	G	04
(	6	05
X	·	06
=	—	07
$\sqrt{x}$	1	08
sto	2	09
)	6	10
—	F	11
X	·	12
(	6	13
rcl	5	14
—	F	15
+	E	16
#	3	17
1	1	18
8	8	19
0	0	20
—	F	21
▼	A	22
gin	1	23
1	1	24
5	5	25
#	3	26
9	9	27
0	0	28
=	—	29
▼	A	30
D→R	3	31
tan	9	32
)	6	33
=	—	34
stop	0	35

HYPERBOLIC FUNCTIONS

If all the hyperbolic functions are likely to be required, use the 'gudermannian' program on page 21 . For the individual functions, the following can be used:

Sinh x

Execution:

x / RUN / sinh x

Range:

$-227{\cdot}95 \leqslant x \leqslant 230{\cdot}25$

(out-of-range may give wrong result without E)

▼	A	00
e^x	4	01
—	F	02
(	6	03
÷	G	04
)	6	05
÷	G	06
#	3	07
2	2	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
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HYPERBOLIC FUNCTIONS

Cosh x

Execution:

x / RUN / cosh x

Range:

$-227{\cdot}95 \leqslant x \leqslant 230{\cdot}25$

(out-of-range may give wrong result without E)

▼	A	00
e^x	4	01
+	E	02
(	6	03
÷	G	04
)	6	05
÷	G	06
#	3	07
2	2	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
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HYPERBOLIC FUNCTIONS

Tanh x

Execution:

x / RUN / tanh x

Range:

|x| ⩽ 113·97
(out-of-range may give wrong result without E)

+	E	00
=	–	01
▼	A	02
e^x	4	03
+	E	04
#	3	05
1	1	06
÷	G	07
+	E	08
–	F	09
#	3	10
1	1	11
–	F	12
=	–	13
stop	0	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
		21
		22
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		34
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HYPERBOLIC FUNCTIONS

Sech x

Execution:

x / RUN / sech x

Range:

|x| ⩽ 227·95

▼	A	00
e^x	4	01
+	E	02
(	6	03
÷	G	04
)	6	05
÷	G	06
+	E	07
=	–	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
		17
		18
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		31
		32
		33
		34
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HYPERBOLIC FUNCTIONS

Cosech x

Execution:

x / RUN / cosech x

Range:

$1{\cdot}0017 \times 10^{-4} \leqslant |x| \leqslant 227{\cdot}95$
($|x| > 227{\cdot}95$ may give wrong result without E)

▼	A	00
e^x	4	01
—	F	02
(	6	03
÷	G	04
)	6	05
÷	G	06
+	E	07
=	—	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
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		30
		31
		32
		33
		34
		35

HYPERBOLIC FUNCTIONS

Coth x

Execution:

x / RUN / coth x

Range:

$1{\cdot}0016 \times 10^{-4} \leqslant |x| \leqslant 113{\cdot}97$
(out-of-range may give wrong result without E)

+	E	00
=	—	01
▼	A	02
e^x	4	03
—	F	04
#	3	05
1	1	06
÷	G	07
+	E	08
+	E	09
#	3	10
1	1	11
=	—	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
0	0	17
		18
		19
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HYPERBOLIC FUNCTIONS

All the hyperbolic functions

▼	A	00
e^x	4	01
+	E	02
#	3	03
1	1	04
÷	G	05
+	E	06
—	F	07
#	3	08
1	1	09
—	F	10
=	—	11
▼	A	12
arctan	9	13
+	E	14
=	—	15
sto	2	16
tan	9	17
stop	0	18
÷	G	19
=	—	20
stop	0	21
rcl	5	22
cos	8	23
÷	G	24
=	—	25
stop	0	26
÷	G	27
=	—	28
stop	0	29
rcl	5	30
sin	7	31
stop	0	32
÷	G	33
=	—	34
stop	0	35

Execution:

x / RUN / sinh x / RUN / cosech x / RUN / cosh x / RUN / sech x / RUN / tanh x / RUN / coth x /

Range:

$1{\cdot}0017 \times 10^{-4} \leqslant |x| \leqslant 7{\cdot}8566$

HYPERBOLIC FUNCTIONS

The gudermannian program

Enables all the hyperbolic functions to be calculated with suitable execution sequences.

Formulae:

$$gd\,x = 2 \arctan \tanh \frac{x}{2}$$

sinh x = tan gdx

cosech x = cot gdx

cosh x = sec gdx

sech x = cos gdx

tanh x = sin gdx

coth x = cosec gdx

Execution:

x / RUN / gdx / ▲▼ / sin / tanh x / ÷ / = / coth x

x / RUN / gdx / ▲▼ / cos / sech x / ÷ / = / cosh x

x / RUN / gdx / ▲▼ / tan / sinh x / ÷ / = / cosech x

This program can be used inside parentheses and does not affect memory.

Accuracy is less than that of individual hyperbolic function programs.

Range:

$|x| \leqslant 227{\cdot}95$ for gdx

$|x| \leqslant 7{\cdot}8566$ for hyperbolic functions

▼	A	00
e^x	4	01
+	E	02
#	3	03
1	1	04
÷	G	05
+	E	06
—	F	07
#	3	08
1	1	09
—	F	10
=	—	11
▼	A	12
arctan	9	13
+	E	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
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		32
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INVERSE HYPERBOLIC FUNCTIONS

All the inverse hyperbolic functions can be obtained from the following program.

Execution:

▲▼ / ▲▼ / goto / 1 / 2 / x / RUN* / $\sinh^{-1}x$
or
x / RUN / $+\cosh^{-1}x$ / RUN / $-\cosh^{-1}x$
or
▲▼ / ▲▼ / goto / 2 / 0 / x / RUN / $\tanh^{-1}x$
or
▲▼ / ▲▼ / goto / 1 / 3 / x / RUN* / $\mathrm{cosech}^{-1}x$
or
▲▼ / ▲▼ / goto / 0 / 1 / x / RUN / $+\mathrm{sech}^{-1}x$ / RUN / $-\mathrm{sech}^{-1}x$
or
▲▼ / ▲▼ / goto / 1 / 9 / x / RUN / $\coth^{-1}x$

* For *negative* x press / RUN / a second time when evaluating $\sinh^{-1}x$ and $\mathrm{cosech}^{-1}x$ to get the correct answer.

Range:

$\sinh^{-1}x$	$10^{-49} \leqslant \lvert x\rvert \leqslant 577{\cdot}35$
$\cosh^{-1}x$	$1 \leqslant x \leqslant 3162{\cdot}2$ No E if x –ve
$\tanh^{-1}x$	$-0{\cdot}99999 \leqslant x \leqslant 0{\cdot}99999$
$\mathrm{cosech}^{-1}x$	$0{\cdot}001732 \leqslant \lvert x\rvert \leqslant 10^{49}$
$\mathrm{sech}^{-1}x$	$3{\cdot}162278 \times 10^{-4} \leqslant x \leqslant 1$ No E if x –ve
$\coth^{-1}x$	$1{\cdot}0001 \leqslant \lvert x\rvert \leqslant 10^{99}$

÷	G	00
X	·	01
–	F	02
+	E	03
#	3	04
1	1	05
=	–	06
√x	1	07
▼	A	08
goto	2	09
2	2	10
0	0	11
÷	G	12
X	·	13
+	E	14
#	3	15
1	1	16
=	–	17
√x	1	18
÷	G	19
–	F	20
+	E	21
#	3	22
1	1	23
÷	G	24
+	E	25
–	F	26
#	3	27
1	1	28
=	–	29
√x	1	30
ln	4	31
stop	0	32
–	F	33
=	–	34
stop	0	35

MODULO ARITHMETIC ('Clock Arithmetic')

Base 7 is used as an example.

The program completes a calculation and works out the remainder when the result is divided by 7. Neither the brackets nor the memory are used, so that the operation of / RUN / is exactly that of / = / .

For other bases, insert the base at steps 03, 11 and 14. Change the address at steps 18 and 19 to 14 if a two digit base is used, 16 for a three digit base, etc.

Execution may take a long time if very large numbers are involved.

Example:

/ 3 / X / 5 / RUN / 1 / + / 8 / RUN / 2 / etc.

–	F	00
+	E	01
#	3	02
7	7	03
–	F	04
▼	A	05
gin	1	06
0	0	07
0	0	08
–	F	09
#	3	10
7	7	11
+	E	12
#	3	13
7	7	14
=	–	15
▼	A	16
gin	1	17
1	1	18
2	2	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

PRIME FACTORISATION

To find the prime factors of a number N.

Pre-execution:

2 / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 / C/CE /

Execution:

N / RUN / a_1 / RUN / N_1 / RUN / a_2 / RUN / N_2 / · · · / a_r / RUN / 1

where
$a_1, a_2, a_3, \cdots a_r$ are the prime factors of N
and
$N_1, N_2, \cdots$ are the residues defined by

$$N_1 = \frac{N}{a_1}, \quad N_2 = \frac{N}{a_1 a_2}, \quad N_3 = \frac{N}{a_1 a_2 a_3}, \text{ etc.}$$

Pressing / RUN / after 1 has been displayed will cause the machine to go into an infinite loop.

Warning: Long execution times are possible for large values of N or for numbers with large prime factors.

÷.	G	00
(	6	01
—	F	02
+	E	03
rcl	5	04
—	F	05
▼	A	06
gin	1	07
0	0	08
2	2	09
=	—	10
▼	A	11
gin	1	12
2	2	13
4	4	14
rcl	5	15
stop	0	16
)	6	17
=	—	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
rcl	5	24
+	E	25
#	3	26
1	1	27
=	—	28
sto	2	29
#	3	30
1	1	31
=	—	32
)	6	33
=	—	34
=	—	35

PRIME NUMBER TESTING

To find whether a number n is prime, choose any integer $m \geqslant \sqrt{n}$.

Then use the execution sequence:

n / RUN / m / RUN /

The result will be the largest number less than or equal to m which divides n. If the result is 1 then n is prime.

To test another number, pre-execute with:

/ ▲▼ / ▲▼ / goto / 0 / 0 /

Note: Long execution times are possible for large numbers.

sto	2	00
stop	0	01
▼	A	02
MEx	5	03
+	E	04
(	6	05
—	F	06
+	E	07
rcl	5	08
—	F	09
▼	A	10
gin	1	11
0	0	12
6	6	13
=	—	14
▼	A	15
gin	1	16
2	2	17
1	1	18
rcl	5	19
stop	0	20
rcl	5	21
—	F	22
#	3	23
1	1	24
=	—	25
sto	2	26
#	3	27
0	0	28
=	—	29
)	6	30
▼	A	31
goto	2	32
0	0	33
4	4	34
		35

FACTORIALS

Execution:

n / RUN / n!

Restriction:

$1 \leqslant n \leqslant 69$

Note: The program may be used within brackets. It does, however, use the memory. Thus, to calculate

$$\frac{15!}{6!\ 10!}$$

a possible execution sequence is:

15 / RUN / ÷ / ▲▼ / (/ 10 / RUN / ▲▼ /) / ÷ / ▲▼ / (/ 6 / RUN / ▲▼ /) / = / answer

sto	2	00
−	F	01
#	3	02
2	2	03
+	E	04
▼	A	05
gin	1	06
2	2	07
1	1	08
#	3	09
1	1	10
X	·	11
▼	A	12
MEx	5	13
=	−	14
▼	A	15
MEx	5	16
▼	A	17
goto	2	18
0	0	19
1	1	20
=	−	21
rcl	5	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
0	0	27
		28
		29
		30
		31
		32
		33
		34
		35

FACTORIALS OF LARGE NUMBERS

This program calculates ln (n!) for n greater than about 25.

Reasonably accurate results are given for n greater than 10.

(The program uses Stirling's approximation, $n! \triangleq \sqrt{2\pi n}\ e^{-n}\ n^n$)

Execution:

n / RUN / ln (n!)

sto	2	00
+	E	01
#	3	02
·	A	03
5	5	04
X	·	05
(	6	06
rcl	5	07
ln	4	08
)	6	09
−	F	10
rcl	5	11
+	E	12
#	3	13
·	A	14
9	9	15
1	1	16
8	8	17
9	9	18
=	−	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

THE GAMMA AND PI FUNCTIONS

$\Gamma(n+1) = \pi(n) = n!$ when n is an integer

$\Gamma(x+1) = x\,\Gamma(x)$ for $x > 0$

$\Gamma(0)$ is undefined $\pi(0) = \Gamma(1) = 1$

$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$ $\pi(\tfrac{1}{2}) = \tfrac{1}{2}\sqrt{\pi}$

$\Gamma(1) = 1$ $\pi(1) = \Gamma(2) = 1$

By interpolation:

$$\Gamma(n+\delta) \simeq (n + \tfrac{1}{2}\delta - \tfrac{1}{2})^{\delta}\,\Gamma(n) \qquad 0 \leqslant \delta \leqslant 1$$

$$\therefore \pi(\delta) \simeq (n + \tfrac{1}{2}\delta - \tfrac{1}{2})^{\delta} \prod_{r=1}^{n-1} \frac{(r)}{r+\delta} \qquad 0 \leqslant \delta \leqslant 1$$

$$\Gamma(\delta) = \frac{\pi(\delta)}{\delta} \simeq \frac{(n + \tfrac{1}{2}\delta - \tfrac{1}{2})^{\delta}}{\delta} \prod_{r=1}^{n} \frac{r}{(r+\delta)}$$

$0 \leqslant \delta \leqslant 1$

n should be suitably large for the accuracy required.

n = 20 gives high accuracy

n = 5 gives reasonable accuracy for most purposes

e.g. $\pi(\tfrac{1}{2}) = \tfrac{1}{2}\sqrt{\pi} = 0{\cdot}8862269$

n = 5 gives $\pi(\tfrac{1}{2}) \simeq 0{\cdot}885547$

n = 20 gives $\pi(\tfrac{1}{2}) \simeq 0{\cdot}8861174$

Execution:

▲▼ / ▲▼ / goto / 0 / 0 /

n / RUN / δ / RUN / n − 1 / RUN / n − 2 / RUN / · · · / 2 / RUN / 1 / RUN / π (δ) / ▲▼ / ▲▼ / goto / 3 / 2 / RUN / Γ (δ)

+	E	00
+	E	01
stop	0	02
sto	2	03
−	F	04
#	3	05
1	1	06
÷	G	07
#	3	08
2	2	09
=	—	10
ln	4	11
X	·	12
rcl	5	13
=	—	14
▼	A	15
e^x	4	16
÷	G	17
(	6	18
stop	0	19
÷	G	20
rcl	5	21
÷	G	22
+	E	23
#	3	24
1	1	25
=	—	26
)	6	27
▼	A	28
goto	2	29
1	1	30
7	7	31
rcl	5	32
)	6	33
=	—	34
stop	0	35

FIBONACCI NUMBERS

Each number in the sequence is the sum of the previous two.

Execution:

C/CE / RUN / F_1 / RUN / F_2 / RUN / · · ·

sto	2	00
#	3	01
1	1	02
+	E	03
stop	0	04
▼	A	05
MEx	5	06
▼	A	07
goto	2	08
0	0	09
3	3	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

NUMBER BASE CONVERSIONS

Decimal to binary (fractions)

Given a decimal x, $0 \leqslant x \leqslant 1$, this program calculates the binary expansion of x to any number of places.

Suppose $x = 0{\cdot}d_1 d_2 \ldots$ (binary)

Execution:

x / RUN / d_1 / RUN / d_2 / RUN / d_3 / · · ·

To calculate the expansion of another decimal y, press

/C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0 / y / RUN / · · · etc.

Notes:

1. To convert decimal integers to binary use the program on page 31.
2. No program for converting decimal fractions to bases other than 2 is provided.

sto	2	00
#	3	01
1	1	02
=	–	03
▼	A	04
MEx	5	05
–	F	06
(	6	07
rcl	5	08
÷	G	09
#	3	10
2	2	11
=	–	12
sto	2	13
)	6	14
–	F	15
(	6	16
▼	A	17
gin	1	18
2	2	19
4	4	20
#	3	21
1	1	22
+	E	23
#	3	24
0	0	25
X	·	26
stop	0	27
rcl	5	28
)	6	29
+	E	30
rcl	5	31
▼	A	32
goto	2	33
0	0	34
6	6	35

NUMBER BASE CONVERSIONS

Decimal integer to base m

This program expresses any integer in any base.

Suppose $x = a_1 \cdots a_r$ in base m.

Execution:

m / RUN / x / RUN / a_r / RUN / a_{r-1} / · · · / a_1 / RUN / m

Note that the digits are produced in reverse order and that the machine tells you that all the digits have been shown by displaying the base m.

The sequence can be repeated for a new x and/or m. If the same m is required there is no need to re-enter it because it is already in the display.

Note: To convert decimal fractions to base 2, use the program on page 30.

sto	2	00
stop	0	01
–	F	02
(	6	03
+	E	04
#	3	05
1	1	06
–	F	07
+	E	08
rcl	5	09
–	F	10
▼	A	11
gin	1	12
0	0	13
7	7	14
+	E	15
rcl	5	16
–	F	17
#	3	18
1	1	19
=	–	20
)	6	21
stop	0	22
÷	G	23
rcl	5	24
–	F	25
–	F	26
▼	A	27
gin	1	28
0	0	29
2	2	30
=	–	31
rcl	5	32
stop	0	33
=	–	34
=	–	35

NUMBER BASE CONVERSIONS

Binary to decimal (integers)

+	E	00
+	E	01
stop	0	02
▼	A	03
goto	2	04
0	0	05
0	0	06
		07
		08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Binary is $a_n \cdots a_o$

Execution:

a_n / RUN / a_{n-1} / RUN / · · · / a_o / = / answer

NUMBER BASE CONVERSIONS

Binary fraction to decimal

#	3	00
1	1	01
=	–	02
sto	2	03
(	6	04
stop	0	05
▼	A	06
MEx	5	07
÷	G	08
#	3	09
2	2	10
X	·	11
▼	A	12
MEx	5	13
)	6	14
+	E	15
▼	A	16
goto	2	17
0	0	18
4	4	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

If number is:

$0 \cdot b_1 b_2 \cdots b_k$

Execution:

RUN / b_1 / RUN / b_2 / · · · / b_k / RUN / answer

At each stage the answer so far is displayed.

Fraction base m to decimal

Exactly the same except / 2 / at step 10 is replaced by the appropriate base.

NUMBER BASE CONVERSIONS

Binary to decimal (integers, fractions or mixed numbers)

+	E	00
+	E	01
stop	0	02
—	F	03
—	F	04
▼	A	05
gin	1	06
0	0	07
0	0	08
sto	2	09
=	—	10
#	3	11
1	1	12
=	—	13
▼	A	14
MEx	5	15
+	E	16
(	6	17
stop	0	18
▼	A	19
MEx	5	20
÷	G	21
#	3	22
2	2	23
X	·	24
▼	A	25
MEx	5	26
)	6	27
▼	A	28
goto	2	29
1	1	30
6	6	31
		32
		33
		34
		35

Binary is $a_n \ldots a_o \cdot b_1 \ldots b_m$

Execution:

C/CE / RUN / a_n / RUN / a_{n-1} / RUN / · · · / RUN / a_o / — / RUN / b_1 / RUN / b_2 / · · · / b_m / RUN / answer

Notes:

1. The / — / corresponding to the 'decimal' point must be entered even if the number is an integer.
2. The correct answer will be given if:

   $a_n = 1$

   $n \geqslant 1$

   $a_o = 1$ or 0

   or just $\cdot b_1 \cdots$ (· entered as —)

To re-use:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

NUMBER BASE CONVERSIONS

Base m to decimal (integers)

sto	2	00
stop	0	01
X	·	02
rcl	5	03
+	E	04
▼	A	05
goto	2	06
0	0	07
1	1	08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Number is $a_n \; a_{n-1} \cdots a_o$

Execution:

m / RUN / a_n / RUN / a_{n-1} / RUN / · · · / a_o / = / answer

To re-use with same m:

C/CE / RUN / a'_n · · ·

NUMBER BASE CONVERSIONS

Base m to decimal (integers, fractions or mixed numbers)

Number is: $a_n \ldots a_o \cdot b_1 \ldots b_p$

Example:

m = 7

Execution:

C/CE / RUN / a_n / RUN / a_{n-1} / · · · / RUN / a_o / – / RUN / b_1 / RUN / b_2 / · · · / RUN / b_p / RUN / answer

Notes:

1. Insert value of m at 02 and 25.
2. If two digit base is used, insert at 02, 03, move the next 22 steps down one, insert the base again at 26, 27, and substitute / 1 / 9 / for / 1 / 8 / in the last two steps.

X	·	00
#	3	01
7	7	02
+	E	03
stop	0	04
–	F	05
–	F	06
▼	A	07
gin	1	08
0	0	09
0	0	10
sto	2	11
=	–	12
#	3	13
1	1	14
=	–	15
▼	A	16
MEx	5	17
+	E	18
(	6	19
stop	0	20
▼	A	21
MEx	5	22
÷	G	23
#	3	24
7	7	25
X	·	26
▼	A	27
MEx	5	28
)	6	29
▼	A	30
goto	2	31
1	1	32
8	8	33
		34
		35

SERIES

Natural numbers

$$(1 + 2 + \cdots + n) = \frac{1}{2}n(n + 1)$$

Execution:

n / RUN / sum

+	E	00
(	6	01
X	·	02
)	6	03
÷	G	04
#	3	05
2	2	06
=	–	07
stop	0	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

SERIES

Squares of natural numbers

sto	2	00
+	E	01
+	E	02
#	3	03
3	3	04
X	·	05
rcl	5	06
+	E	07
#	3	08
1	1	09
X	·	10
rcl	5	11
÷	G	12
#	3	13
6	6	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

$$(1+4+9+\cdots+n^2)=\frac{1}{6}n(n+1)(2n+1)$$

Execution:

n / RUN / sum

SERIES

Cubes of natural numbers

+	E	00
(	6	01
X	·	02
)	6	03
X	·	04
÷	G	05
#	3	06
4	4	07
=	—	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

$$(1+8+27+\cdots n^3)=\frac{1}{4}n^2(n+1)^2$$

Execution:

n / RUN / sum

ARITHMETIC SERIES

First term = a
Common difference = d
N terms

$$\text{sum} = N\left(a + \frac{(N-1)d}{2}\right)$$

Execution:

a / RUN / N / RUN / d / RUN / sum

+	E	00
(	6	01
stop	0	02
sto	2	03
—	F	04
#	3	05
1	1	06
÷	G	07
#	3	08
2	2	09
X	·	10
stop	0	11
)	6	12
X	·	13
rcl	5	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

ARITHMETIC SERIES

First term = a
Last term = l
N terms

$$\text{sum} = \frac{N(a + l)}{2}$$

Execution:

a / RUN / l / RUN / N / RUN / sum

+	E	00
stop	0	01
÷	G	02
#	3	03
2	2	04
X	·	05
stop	0	06
=	—	07
stop	0	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

GEOMETRIC SERIES

$$S = a + ar + \cdots + ar^{N-1} = \frac{a(1-r^n)}{(1-r)}$$

First term = a

Common ratio = r

N terms

Restrictions:

$r > 0, \; r \neq 1$

Execution:

a / RUN / r / RUN / N / RUN / sum

÷	G	00
(	6	01
stop	0	02
sto	2	03
—	F	04
#	3	05
1	1	06
=	—	07
)	6	08
X	·	09
(	6	10
rcl	5	11
ln	4	12
X	·	13
stop	0	14
=	—	15
▼	A	16
e^x	4	17
—	F	18
#	3	19
1	1	20
=	—	21
)	6	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

INFINITE GEOMETRIC SERIES

$$S = a + ar + ar^2 + \cdots = \frac{a}{1-r}$$

Restriction:

$|r| < 1$

Execution:

a / RUN / r / RUN / sum

÷	G	00
(	6	01
#	3	02
1	1	03
—	F	04
stop	0	05
)	6	06
=	—	07
stop	0	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

ARITHMETIC – GEOMETRIC SERIES (infinite)

—	F	00
#	3	01
1	1	02
—	F	03
÷	G	04
X	·	05
(	6	06
—	F	07
#	3	08
1	1	09
X	·	10
stop	0	11
+	E	12
stop	0	13
)	6	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

$S = a + (a + d)r + (a + 2d)r^2 + \cdots + (a + nd)r^n + \cdots$

$$= \frac{a + \frac{dr}{1 - r}}{1 - r}$$

Restriction:

$|r| < 1$

Execution:

r / RUN / d / RUN / a / RUN / sum

SUMMING SERIES IN GENERAL

$\sum_{1}^{N} a(n)$, some function a.

Examples:

1. $1 + 4 + 9 + \cdots + N^2$ $\qquad a(n) = n^2$
2. $\left(1 + \frac{1}{1}\right) + \left(8 + \frac{1}{4}\right) + \cdots + \left(N^3 + \frac{1}{N^2}\right)$

   $a(n) = n^3 + \frac{1}{n^2}$ $\qquad$ etc.

Write a program segment which evaluates a(n) when n is in memory; parentheses may not be used. The segment may be up to 15 steps long, any final / = / stop / being omitted. Fill up any unused steps with / = / · · · / = /.

Examples for above:

1. n^2 $\qquad$ rcl / X /
2. $n^3 + \frac{1}{n^2}$ $\qquad$ write as $(n^5 + 1) \div n^2$

   rcl / X / X / X / rcl / + / # / 1 / ÷ / rcl / rcl / = / = / = /

   Then use the program as shown.

Pre-execution:

Clear memory with C/CE / ▲▼ / sto /

Execution:

N / RUN / a(1) + a(2) + · · · + a(n)

=	—	00
▼	A	01
MEx	5	02
+	E	03
(	6	04
		05
Y		06
O		07
U		08
R		09
		10
S		11
E		12
G		13
M		14
E		15
N		16
T		17
		18
		19
)	6	20
=	—	21
▼	A	22
MEx	5	23
—	F	24
#	3	25
1	1	26
—	F	27
—	F	28
▼	A	29
gin	1	30
0	0	31
0	0	32
=	—	33
rcl	5	34
stop	0	35

SERIES

$a(x_1) + a(x_2) + \cdots + a(x_n)$

Write a program segment to evaluate $a(x_i)$ without using parentheses; the memory may be used.

Then use the following program:

/ (/ stop / · · · seg · · · /) / + / ▼/ goto / 0 / 0 /

Execution:

RUN / x_1 / RUN / x_2 / · · · / x_n / RUN / sum

At each step the sum so far is displayed.

Example:

To find $\Sigma \tan\left(x^2 + \frac{1}{x}\right)$

Express $x^2 + \frac{1}{x}$ as $\frac{x^3 + 1}{x}$

Program segment is then:

/ sto / X / X / rcl / + / # / 1 / ÷ / rcl / = / tan /

and so program is as shown.

The segment may be up to 32 steps long, by omitting / ▼/ goto / 0 / 0 / at the end and filling any empty steps with / = /.

(	6	00
stop	0	01
sto	2	02
X	·	03
X	·	04
rcl	5	05
+	E	06
#	3	07
1	1	08
÷	G	09
rcl	5	10
=	—	11
tan	9	12
)	6	13
+	E	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

HARMONIC ADDITION

Resistors in parallel, capacitors in series, lenses in series, etc.

$$\frac{1}{x} = \frac{1}{x_1} + \cdots + \frac{1}{x_n}$$

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN / x

At each step the harmonic sum so far is displayed.

÷	G	00
+	E	01
(	6	02
÷	G	03
=	—	04
stop	0	05
÷	G	06
)	6	07
▼	A	08
goto	2	09
0	0	10
1	1	11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

PYTHAGOREAN ADDITION

Geometry, electricity

×	·	00
+	E	01
(	6	02
√x	1	03
stop	0	04
×	·	05
)	6	06
▼	A	07
goto	2	08
0	0	09
1	1	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
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		31
		32
		33
		34
		35

$x = \sqrt{x_1^2 + \cdots + x_n^2}$

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN / x

At each step the intermediate result $\sqrt{x_1^2 + \cdots + x_i^2}$ is displayed.

ARITHMETIC MEAN

×	·	00
(	6	01
#	3	02
1	1	03
=	–	04
sto	2	05
)	6	06
+	E	07
(	6	08
stop	0	09
÷	G	10
rcl	5	11
)	6	12
÷	G	13
(	6	14
#	3	15
1	1	16
+	E	17
rcl	5	18
÷	G	19
▼	A	20
MEx	5	21
)	6	22
▼	A	23
goto	2	24
0	0	25
7	7	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Pre-execution:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN /
arithmetic mean

At each stage the arithmetic mean so far is displayed.

GEOMETRIC MEAN

ln	4	00
X	·	01
(	6	02
#	3	03
1	1	04
=	–	05
sto	2	06
)	6	07
+	E	08
(	6	09
▼	A	10
e^x	4	11
stop	0	12
ln	4	13
÷	G	14
rcl	5	15
)	6	16
÷	G	17
(	6	18
#	3	19
1	1	20
+	E	21
rcl	5	22
÷	G	23
▼	A	24
MEx	5	25
)	6	26
▼	A	27
goto	2	28
0	0	29
8	8	30
		31
		32
		33
		34
		35

Pre-execution:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN /
geometric mean

At each stage the geometric mean so far is displayed.

HARMONIC MEAN

÷	G	00
X	·	01
(	6	02
#	3	03
1	1	04
=	–	05
sto	2	06
)	6	07
+	E	08
(	6	09
÷	G	10
=	–	11
stop	0	12
÷	G	13
÷	G	14
rcl	5	15
)	6	16
÷	G	17
(	6	18
#	3	19
1	1	20
+	E	21
rcl	5	22
÷	G	23
▼	A	24
MEx	5	25
)	6	26
▼	A	27
goto	2	28
0	0	29
8	8	30
		31
		32
		33
		34
		35

$$\frac{1}{H}=\frac{1}{n}\left(\frac{1}{x_1}+\cdots+\frac{1}{x_n}\right)$$

Pre-execution:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

Execution:

x_1 / RUN / x_2 / RUN / · · · / x_n / RUN /
harmonic mean

At each stage the harmonic mean so far is displayed.

ROOT MEAN SQUARE

$$R = \frac{\sqrt{(x_1^2 + \cdots + x_n^2)}}{n}$$

Pre-execution:

C/CE / C/CE / ▲▼ / ▲▼ / goto / 0 / 0

Execution:

x_1 / RUN / x_2 / · · · / x_n / RUN /
root-mean-square

At each stage the r.m.s. so far is displayed.

X	·	00
X	·	01
(	6	02
#	3	03
1	1	04
=	—	05
sto	2	06
)	6	07
+	E	08
(	6	09
√x	1	10
stop	0	11
X	·	12
÷	G	13
rcl	5	14
)	6	15
÷	G	16
(	6	17
#	3	18
1	1	19
+	E	20
rcl	5	21
÷	G	22
▼	A	23
MEx	5	24
)	6	25
▼	A	26
goto	2	27
0	0	28
8	8	29
		30
		31
		32
		33
		34
		35

QUADRATIC EQUATIONS

$ax^2 + bx + c = 0$

Roots x_1, x_2 if real

R ± iI if complex

Execution:

a / RUN / b / RUN / c / RUN /
- x_1 / RUN / x_2 / RUN / RUN / C/CE / C/CE / if roots are real
- I* / C/CE / RUN / R / if roots are complex

* error symbol displayed

After the sequence a / RUN / b / RUN / c / RUN / the display shows *either* (if the roots are real) the larger real root with no error indication *or* (if the roots are complex) the imaginary part and the error symbol. Continue with the appropriate execution sequence.

The error symbol will tell you whether the roots are complex. The sequence / RUN / RUN / C/CE / shown above after (x_2) is necessary before entering a new equation to be solved.

+	E	00
÷	G	01
—	F	02
X	·	03
sto	2	04
stop	0	05
=	—	06
▼	A	07
MEx	5	08
X	·	09
stop	0	10
+	E	11
+	E	12
(	6	13
rcl	5	14
X	·	15
)	6	16
+	E	17
▼	A	18
gin	1	19
3	3	20
2	2	21
√x	1	22
▼	A	23
MEx	5	24
—	F	25
stop	0	26
rcl	5	27
—	F	28
rcl	5	29
=	—	30
stop	0	31
√x	1	32
stop	0	33
rcl	5	34
stop	0	35

CUBIC EQUATIONS
by an iterative method

$ax^3 + bx^2 + cx + d = 0$

Formula:

$$x_{k+1} = \frac{2ax_k^3 + bx_k^2 - d}{3ax_k^2 + 2bx_k + c}$$ (based on Newton-Raphson method)

(Fill in your own values of 2a, b, d, etc.; if any of these are negative change the + or — preceding them to — or +)

Execution:

Choose any starting value x_o, say $-\frac{d}{c}$

x_o / RUN / x_1 / RUN / x_2 / · · ·

If the sequence converges, the limit will solve the equation.

If the sequence does not converge, try a new starting value.

The sequence will usually converge to the root closest to the starting value and so by trying different starting values all the roots should be obtained.

* where $a_1 a_2$ is the two digit number 3a; if $3a < 10$ then enter $a_1 = 0$ and a_2 as the value of 3a. Similarly $b_1 b_2$ is 2b.

sto	2	00
X	·	01
#	3	02
a	a	03
+	E	04
+	E	05
#	3	06
b	b	07
X	·	08
rcl	5	09
X	·	10
rcl	5	11
—	F	12
#	3	13
d	d	14
÷	G	15
(	6	16
#	3	17
* a_1	a_1	18
a_2	a_2	19
X	·	20
rcl	5	21
+	E	22
#	3	23
* b_1	b_1	24
b_2	b_2	25
X	·	26
rcl	5	27
+	E	28
#	3	29
c	c	30
=	—	31
)	6	32
=	—	33
stop	0	34
=	—	35

POLYNOMIALS

To evaluate

$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_o = p(x)$

Execution:

x / RUN / a_n / RUN / a_{n-1} / · · · / a_1 / RUN / a_o / = / result

To use again: (with different x)

▲▼ / ▲▼ / goto / 0 / 0 / before execution

Notes:

1. The individual results after each / RUN / are the coefficients of the polynomial q(x) where q(t) = p(t) / (t — x).
2. If p(x) = 0, x is a root and q(x) is the quotient polynomial which can be solved for other roots of p(x).

sto	2	00
stop	0	01
X	·	02
rcl	5	03
+	E	04
▼	A	05
goto	2	06
0	0	07
1	1	08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
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POLYNOMIALS

To write a program to evaluate the same polynomial repeatedly

Example:

$p(x) = 5x^4 + 8x^3 - 3x^2 + 4{\cdot}2x + 1$

Method:

Express as $⟦[(5x + 8)x - 3]x + 4{\cdot}2⟧x + 1$

Execution:

x / RUN / p(x) / y / RUN / p(y) · · · etc.

Note: If a coefficient is zero omit it together with the — or + sign preceding it. If the leading coefficient is 1, it may be omitted together with the multiplication sign which precedes it. See over for example.

sto	2	00
X	·	01
#	3	02
5	5	03
+	E	04
#	3	05
8	8	06
X	·	07
rcl	5	08
—	F	09
#	3	10
3	3	11
X	·	12
rcl	5	13
+	E	14
#	3	15
4	4	16
·	A	17
2	2	18
X	·	19
rcl	5	20
+	E	21
#	3	22
1	1	23
=	—	24
stop	0	25
▼	A	26
goto	2	27
0	0	28
0	0	29
		30
		31
		32
		33
		34
		35

POLYNOMIALS

first coefficient = 1, so omitted.

coefficient of x = 0, so omitted.

Example:

To calculate $x^3 + 2x^2 + 3$

sto	2	00
+	E	01
#	3	02
2	2	03
X	·	04
rcl	5	05
X	·	06
rcl	5	07
+	E	08
#	3	09
3	3	10
=	—	11
stop	0	12
▼	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
		20
		21
		22
		23
		24
		25
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DIVISION OF A POLYNOMIAL BY A QUADRATIC

Division of the polynomial

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_o$

by the quadratic divisor

$d(x) = x^2 + mx + n$

gives the quotient polynomial

$q(x) = b_{n-2} x^{n-2} + b_{n-3} x^{n-3} + \cdots + b_1 x + b_o$

with remainder

$r(x) = c_1 x + c_o$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / C/CE / ▲▼ / sto /

Execution:

RUN / n / RUN / m / RUN / a_n / RUN / b_{n-2}
RUN / n / RUN / m / RUN / a_{n-1} / RUN / b_{n-3}
RUN / n / RUN / m / RUN / a_2 / RUN / b_o
RUN / n / RUN / m / RUN / a_1 / RUN / c_1
RUN / n / RUN / m / RUN / a_o / RUN / RUN / m / RUN / 1 / RUN / c_o
/ RUN / RUN / completes execution

Results may be tabulated as below: e.g. to divide $x^6 - 4x^5 + 31x^4 - 96x^3 + 415x^2 - 652x + 1105$ by $x^2 + 2x + 3$:

r	n	m	a_r	b_{r-2}
6	3	2	1	1
5			−4	−6
4			31	40
3			−96	−158
2			415	611
1			−652	$-1400 = c_1$
0			1105	$-728 = c_o$

▼	A	00
MEx	5	01
X	·	02
stop	0	03
+	E	04
(	6	05
stop	0	06
X	·	07
rcl	5	08
)	6	09
—	F	10
stop	0	11
—	F	12
=	—	13
stop	0	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
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SOLVING A POLYNOMIAL

This is an iterative method to find a quadratic factor of a polynomial. When the polynomial has been reduced to quadratic factors, these can be solved to give the real or complex roots of the original polynomial.

Stage 1:

Choose a starting quadratic divisor

$d(x) = x^2 + mx + n$ (say)

Divide p(x) by d(x) to give a quotient q(x) and remainder $r(x) = rx + s$

Stage 2:

Divide q(x) again by d(x) to give a new quotient q′(x) and remainder $r'(x) = tx + u$

Stage 3:

Find the coefficients m′ and n′ of the next iterate of the quadratic divisor using this program

Execution:

u / RUN / t / RUN / m / RUN / n / RUN / D

p / RUN / t / RUN / u / RUN / s / RUN / r / RUN / t / RUN / — / + / n / = / n′

▲▼ / ▲▼ / goto / 2 / 5 / r / RUN / u / RUN / n / X / s / RUN / t / RUN / + / m / = / m′

$D = u^2 + nt^2 - mut$

$$m' = m + \frac{ru + nst}{D}$$

$$n' = n - \frac{rt + s(mt - u)}{D}$$

Re-enter the quadratic divisor program and iterate again with the new values of m′ and n′. Repeat stages 1–3 until the values of m and n converge.

X	·	00
sto	2	01
—	F	02
(	6	03
rcl	5	04
X	·	05
stop	0	06
sto	2	07
X	·	08
stop	0	09
)	6	10
+	E	11
(	6	12
rcl	5	13
X	·	14
X	·	15
stop	0	16
)	6	17
=	—	18
sto	2	19
stop	0	20
X	·	21
stop	0	22
—	F	23
stop	0	24
X	·	25
stop	0	26
+	E	27
(	6	28
stop	0	29
X	·	30
stop	0	31
)	6	32
÷	G	33
rcl	5	34
stop	0	35

NUMERICAL INTEGRATION

Triangular interpolation

$$I = \frac{1}{2}h(y_o + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$$

Execution:

n / RUN / y_o / RUN / y_1 / RUN / y_2 / RUN / · · · / RUN / y_n / RUN / h / RUN / I

–	F	00
#	3	01
1	1	02
=	–	03
sto	2	04
stop	0	05
+	E	06
(	6	07
rcl	5	08
–	F	09
#	3	10
1	1	11
=	–	12
sto	2	13
▼	A	14
gin	1	15
2	2	16
5	5	17
stop	0	18
+	E	19
)	6	20
▼	A	21
goto	2	22
0	0	23
6	6	24
stop	0	25
)	6	26
X	·	27
stop	0	28
÷	G	29
#	3	30
2	2	31
=	–	32
stop	0	33
=	–	34
=	–	35

NUMERICAL INTEGRATION

Simpson's Rule

$$I = \frac{1}{3}h(y_o + 4y_1 + 2y_2 + 4y_3 + \cdots + 4y_{n-1} + y_n)$$

(n must be even)

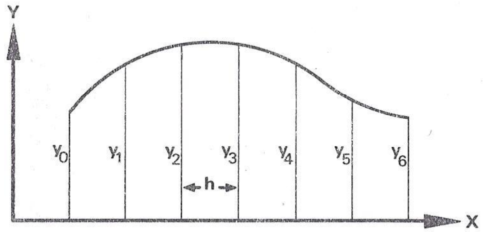
Execution:

n / – / 1 / = / RUN / y_1 / RUN / · · · / y_n / RUN / h / RUN / I

sto	2	00
stop	0	01
+	E	02
(	6	03
stop	0	04
+	E	05
+	E	06
)	6	07
+	E	08
(	6	09
rcl	5	10
–	F	11
#	3	12
2	2	13
=	–	14
sto	2	15
▼	A	16
gin	1	17
2	2	18
7	7	19
stop	0	20
+	E	21
)	6	22
▼	A	23
goto	2	24
0	0	25
2	2	26
stop	0	27
)	6	28
X	·	29
stop	0	30
÷	G	31
#	3	32
3	3	33
=	–	34
stop	0	35

NUMERICAL INTEGRATION

Weddle Formula



$$\text{Integral} = \frac{3h}{10}(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6)$$

Execution:

y_0 / RUN / y_1 / RUN / y_2 / RUN / y_3 / RUN / y_4 / RUN / y_5 / RUN / y_6 / RUN / h_1 / = / integral

+	E	00
(	6	01
stop	0	02
X	·	03
#	3	04
5	5	05
=	—	06
)	6	07
+	E	08
stop	0	09
+	E	10
(	6	11
stop	0	12
X	·	13
#	3	14
6	6	15
=	—	16
)	6	17
+	E	18
stop	0	19
+	E	20
(	6	21
stop	0	22
X	·	23
#	3	24
5	5	25
=	—	26
)	6	27
+	E	28
stop	0	29
X	·	30
#	3	31
·	A	32
3	3	33
X	·	34
stop	0	35

COMPLEX NUMBERS

$z = x + iy$

To find magnitude and argument.

Execution:

If $y = 0$, then $z = |x|$ and arg $z = (0$ if $x \geqslant 0$, π if $x < 0)$

Otherwise, x / RUN / y / RUN / |z| / RUN / arg z

To find x and y given arg z and |z|

$(-\pi \leqslant \arg z \leqslant \pi)$

If arg z is 0, then x = |z| and y = 0

If arg z is π, then x = −|z| and y = 0

Otherwise use polar-cartesian program, execution as follows:

|z| / RUN / arg z / RUN / x / RUN / y

÷	G	00
(	6	01
X	·	02
÷	G	03
stop	0	04
sto	2	05
+	E	06
rcl	5	07
X	·	08
rcl	5	09
=	—	10
√x	1	11
stop	0	12
)	6	13
+	E	14
#	3	15
1	1	16
÷	G	17
#	3	18
2	2	19
=	—	20
√x	1	21
▼	A	22
arccos	8	23
+	E	24
X	·	25
(	6	26
rcl	5	27
X	·	28
÷	G	29
√x	1	30
rcl	5	31
)	6	32
=	—	33
stop	0	34
=	—	35

DETERMINANTS

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Execution:

a_1 / RUN / b_1 / RUN / a_2 / RUN / b_2 / RUN / det

sto	2	00
stop	0	01
X	·	02
stop	0	03
—	F	04
(	6	05
rcl	5	06
X	·	07
stop	0	08
)	6	09
—	F	10
=	—	11
stop	0	12
▲	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

MATRIX MANIPULATION

1. Matrix multiplication (steps 00–11)

AB = C

$$C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Execution:

a_{i1} / RUN / b_{1j} / RUN /

a_{i2} / RUN / b_{2j} / RUN / · · ·

a_{in} / RUN / b_{nj} / RUN / c_{ij}

To restore zero total for next calculation, press C/CE.

Error correction:

For a_{ik}: C/CE / + / a_{ik}

For b_{kj}: ▲▼ /) / C/CE / + / ▲▼ / (/ b_{kj}

2. Back substitution (steps 00–21)

(for AX = B where A is upper triangular)

$$x_{ij} = \frac{\left(b_{ij} - \sum_{k=i+1}^{n} a_{ik} x_{kj}\right)}{a_{ii}}$$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / for each x_{ij}

sto	2	00
(	6	01
stop	0	02
X	·	03
rcl	5	04
)	6	05
+	E	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
#	3	12
0	0	13
—	F	14
stop	0	15
—	F	16
÷	G	17
stop	0	18
=	—	19
sto	2	20
stop	0	21
X	·	22
rcl	5	23
+	E	24
stop	0	25
=	—	26
▼	A	27
goto	2	28
2	2	29
1	1	30
		31
		32
		33
		34
		35

MATRIX MANIPULATION

Execution:

x_{nj} / RUN / a_{in} / RUN / · · · / $x_{i+1,j}$ / RUN / $a_{i,i+1}$ / RUN / $\Sigma a_{ik} x_{kj}$

▲▼ / ▲▼ / goto / 1 / 2 / RUN / b_{ij} / RUN / a_{ii} / RUN / x_{ij}

Error correction:

For x_{kj}: C/CE / + / x_{kj}

For a_{ik}: / ▲▼ /) / C/CE / + / ▲▼ / (/ a_{ik}

For b_{ij}: C/CE / − / b_{ij}

For a_{ii}: C/CE / ÷ / a_{ii}

3. Adding a multiple of row i to row j in the augmented matrix (A/B) (steps 16–30)

$a'_{jk} = a_{jk} + m_{ji} a_{ik}$, $\quad b'_{jk} = b_{jk} + m_{ji} b_{ik}$

where $m_{ji} = -\dfrac{a_{ji}}{a_{ii}}$

Pre-execution (each m_{ji}):

▲▼ / ▲▼ / goto / 1 / 6 / C/CE

Execution:

a_{ji} / RUN / a_{ii} / RUN / m_{ji} error correction: re-run from 16

a_{ik} / RUN / a_{jk} / RUN / a'_{jk} for each k

b_{ik} / RUN / b_{jk} / RUN / b'_{jk} for each k

Note: If m_{ji} is known pre-execution can be ▲▼ / ▲▼ / goto / 1 / 9 / C/CE and first part of execution m_{ji} / RUN / m_{ji}

EQUATION SOLVING

The secant method

In this variant of the Newton-Raphson method for solving the equation f(x) = 0, instead of computing the derivative f′(x) at each stage, an approximation to f′(x) at a point in the vicinity of a root x_r is used.

Stage 1:

Write a program segment to compute f(x) when x is in memory, taking up no more than 27 steps excluding the final / stop /. Enter the program starting at step 01, ending with the sequence / stop / ▼/ goto / 0 / 0 /.

Execution: x / RUN / f(x)

Evaluate f(x) for a range of values in which a root is likely to occur. If $f(x_1)$ and $f(x_2)$ have opposite signs, there is a root between x_1 and x_2.

Stage 2:

Calculate an approximation to the derivative of f(x) as follows:

$f(x_2)$ / − / $f(x_1)$ / ÷ / ▲▼ / (/ x_1 / − / x_2 / ▲▼ /) / = / k $\doteq -f'(x_r)$

Stage 3:

The iteration formula for the secant method is

$$x' = x + \frac{f(x)}{K}$$

where K is a constant approximately equal to the derivative of f(x) at the root. K may be chosen to be equal to k, or may be an integer or a number with fewer digits than k, in which case it should be numerically larger than k.

Note: If the program segment in Stage 1 took 27 steps, there is room for only one digit for K in the following program. (contd. over)

sto	2	00
		01
		02
		03
		04
		05
		06
		07
		08
		09
		10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
(÷)	(F)	28
÷	G	29
#	3	30
K	K	31
+	E	32
rcl	5	33
=	–	34
stop	0	35

Starting at the final / stop / step, press / ▲▼ / LEARN / and enter the sequence:

/ ÷ / # / K / + / rcl / = / stop / for K positive, or

/ — / ÷ / # / K / + / rcl / = / stop / for negative K

The sequence / ▼ / goto / 0 / 0 / or / = / steps may be added at the end.

Execution: x / RUN / x'
/ RUN / x'' · · ·

Repeat until successive values are equal. If convergence is slow, decrease K. If the results diverge, increase K.

If k is a small fraction, the / ÷ / step may be replaced by a / X / step and K taken as the reciprocal of k.

See below for example.

Example:

To solve cos x = x

f(x) = cos x — x

Take $x_1 = \frac{\pi}{2}$, $x_0 = 0$.

Then $$\frac{f(x_0) - f(x_1)}{x_1 - x_0} = \frac{1 + \frac{\pi}{2}}{\frac{\pi}{2}} = 2$$

Program segment is / cos / — / rcl

Guess 1 as starting solution

Execution:

1 / RUN / 0·770223
/ RUN / 0·7440342
/ RUN / 0·7399375
/ RUN / 0·7392705
/ RUN / 0·7391738
/ RUN / 0·7391592
/ RUN / 0·7391519
/ RUN / 0·7391483
/ RUN / 0·7391465
/ RUN / 0·7391456
/ RUN / 0·7391451
/ RUN / 0·7391449
/ RUN / 0·7391448
/ RUN / 0·7391447
/ RUN / 0·7391447

So result is 0·7391447

sto	2	00
cos	8	01
—	F	02
rcl	5	03
÷	G	04
#	3	05
2	2	06
+	E	07
rcl	5	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CIRCLES

Circumference and area

Execution:

radius / RUN / circumference / RUN / area

X	·	00
(	6	01
X	·	02
#	3	03
6	6	04
·	A	05
2	2	06
8	8	07
3	3	08
1	1	09
9	9	10
=	—	11
stop	0	12
)	6	13
÷	G	14
#	3	15
2	2	16
=	—	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CIRCLES

Radius of circle from area

$$r = \sqrt{\frac{A}{\pi}}$$

Execution:

A / RUN / r

÷	G	00
#	3	01
3	3	02
·	A	03
1	1	04
4	4	05
1	1	06
5	5	07
9	9	08
2	2	09
6	6	10
=	—	11
$\sqrt{x}$	1	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
0	0	17
		18
		19
		20
		21
		22
		23
		24
		25
		26
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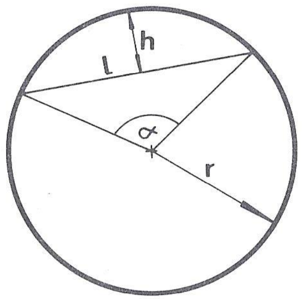
		29
		30
		31
		32
		33
		34
		35

CIRCLES

Area of segment:



Area of segment if h and r are given:

$$\text{Area} = \frac{r^2}{2}(\alpha - \sin\alpha)$$

$$\text{where } \cos\frac{\alpha}{2} = \frac{r - h}{r}$$

Note: the angle α is calculated internally and is not required to be input.

Execution:

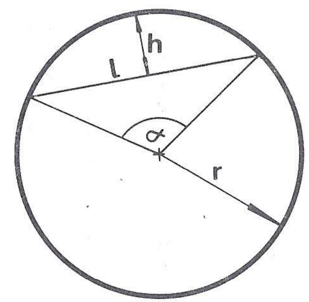
r / RUN / h / RUN / area

Note: limited range, $\alpha < 1{\cdot}57$ radians

sto	2	00
—	F	01
stop	0	02
÷	G	03
rcl	5	04
=	—	05
▼	A	06
arccos	8	07
+	E	08
—	F	09
(	6	10
sin	7	11
)	6	12
×	·	13
(	6	14
rcl	5	15
×	·	16
)	6	17
÷	G	18
#	3	19
2	2	20
=	—	21
stop	0	22
▼	A	23
goto	2	24
0	0	25
0	0	26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CIRCLES

Length of chord



$$l = 2\sqrt{2hr - h^2}$$

Execution:

h / RUN / r / RUN / length

sto	2	00
×	·	01
(	6	02
stop	0	03
+	E	04
—	F	05
rcl	5	06
)	6	07
+	E	08
√x	1	09
=	—	10
stop	0	11
▼	A	12
goto	2	13
0	0	14
0	0	15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
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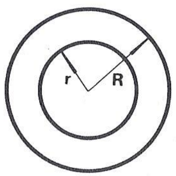
		29
		30
		31
		32
		33
		34
		35

CIRCLES

Area of circular annulus



Area = $\pi(R^2 - r^2)$

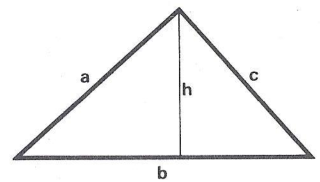
Execution:

R / RUN / r / RUN / area

X	·	00
—	F	01
(	6	02
stop	0	03
X	·	04
)	6	05
X	·	06
#	3	07
3	3	08
·	A	09
1	1	10
4	4	11
1	1	12
5	5	13
9	9	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

TRIANGLES

To find area, given base and height



$$A = \frac{bh}{2}$$

Execution:

b / RUN / h / RUN / area

X	·	00
stop	0	01
÷	G	02
#	3	03
2	2	04
=	—	05
stop	0	06
▼	A	07
goto	2	08
0	0	09
0	0	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
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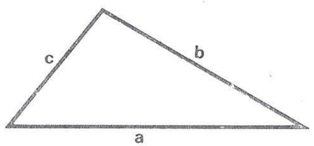
		30
		31
		32
		33
		34
		35

TRIANGLES

To find area, given all three sides



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \left(\frac{a+b+c}{2}\right)$$

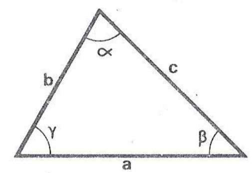
Execution:

a / RUN / b / RUN / c / RUN / b / RUN / a / RUN / area

+	E	00
stop	0	01
+	E	02
stop	0	03
sto	2	04
÷	G	05
#	3	06
2	2	07
X	·	08
(	6	09
▼	A	10
MEx	5	11
—	F	12
rcl	5	13
—	F	14
)	6	15
X	·	16
(	6	17
rcl	5	18
—	F	19
stop	0	20
)	6	21
X	·	22
(	6	23
rcl	5	24
—	F	25
stop	0	26
)	6	27
=	—	28
$\sqrt{x}$	1	29
stop	0	30
▼	A	31
goto	2	32
0	0	33
0	0	34
		35

TRIANGLES

Finding a side, given two angles and a side



$$a = \frac{b \sin \alpha}{\sin \beta}$$

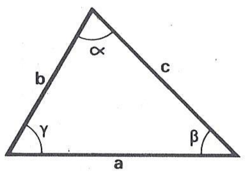
Execution:

α° / RUN / β° / RUN / b / RUN / a

—	F	00
#	3	01
9	9	02
0	0	03
X	·	04
=	—	05
$\sqrt{x}$	1	06
▼	A	07
D→R	3	08
cos	8	09
÷	G	10
(	6	11
stop	0	12
—	F	13
#	3	14
9	9	15
0	0	16
X	·	17
=	—	18
$\sqrt{x}$	1	19
cos	8	20
)	6	21
X	·	22
stop	0	23
=	—	24
stop	0	25
▼	A	26
goto	2	27
0	0	28
0	0	29
		30
		31
		32
		33
		34
		35

TRIANGLES

Length of third side from two sides and included angle



$$a = \sqrt{b^2 + c^2 - 2bc \cos \alpha}$$

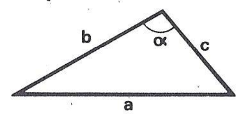
Execution:

b / RUN / c / RUN / α° / RUN / a

sto	2	00
stop	0	01
X	·	02
(	6	03
–	F	04
rcl	5	05
X	·	06
=	–	07
▼	A	08
MEx	5	09
+	E	10
)	6	11
X	·	12
(	6	13
stop	0	14
–	F	15
#	3	16
9	9	17
0	0	18
=	–	19
▼	A	20
D→R	3	21
sin	7	22
+	E	23
#	3	24
1	1	25
=	–	26
)	6	27
+	E	28
rcl	5	29
=	–	30
$\sqrt{x}$	1	31
stop	0	32
=	–	33
=	–	34
=	–	35

TRIANGLES

Finding an angle, given three sides



$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

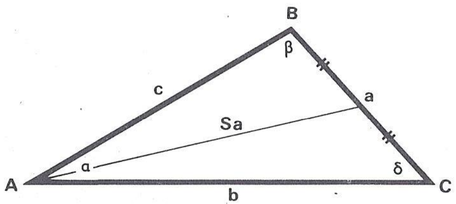
Execution:

a / RUN / b / RUN / c / RUN / α°

÷	G	00
stop	0	01
sto	2	02
X.	·	03
–	F	04
+	E	05
#	3	06
1	1	07
X	·	08
(	6	09
stop	0	10
÷	G	11
rcl	5	12
=	–	13
sto	2	14
÷	G	15
)	6	16
+	E	17
rcl	5	18
÷	G	19
#	3	20
2	2	21
=	–	22
▼	A	23
arcsin	7	24
▼	A	25
R→D	6	26
–	F	27
+	E	28
#	3	29
9	9	30
0	0	31
=	–	32
stop	0	33
=	–	34
=	–	35

TRIANGLES

Length of medians, given lengths of sides



$$S_a = \frac{\sqrt{2(b^2 + c^2) - a^2}}{2}$$

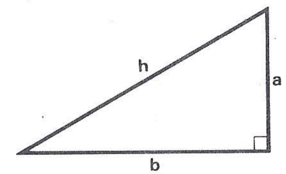
Execution:

b / RUN / c / RUN / a / RUN / S_a

×	·	00
+	E	01
(	6	02
stop	0	03
×	·	04
)	6	05
+	E	06
—	F	07
(	6	08
stop	0	09
×	·	10
)	6	11
÷	G	12
#	3	13
4	4	14
=	—	15
$\sqrt{x}$	1	16
stop	0	17
▼	A	18
goto	2	19
0	0	20
0	0	21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

RIGHT ANGLED TRIANGLES

Length of hypotenuse from other two sides



$$h = \sqrt{a^2 + b^2}$$

Execution:

a / RUN / b / RUN / h

×	·	00
+	E	01
(	6	02
stop	0	03
×	·	04
)	6	05
=	—	06
$\sqrt{x}$	1	07
stop	0	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
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		31
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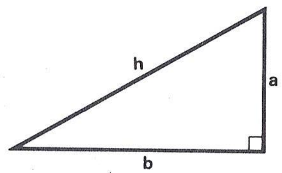
		33
		34
		35

RIGHT ANGLED TRIANGLES

Length of one short side from other two sides



$b = \sqrt{h^2 - a^2}$

Execution:

a / RUN / h / RUN / b

X	·	00
—	F	01
(	6	02
stop	0	03
X	·	04
)	6	05
—	F	06
=	—	07
$\sqrt{x}$	1	08
stop	0	09
▲	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
		17
		18
		19
		20
		21
		22
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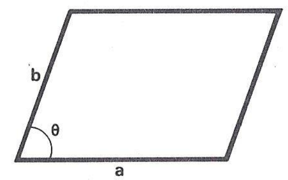
PARALLELOGRAMS



Area = ab sin θ

Execution:

a / RUN / b / RUN / $\theta°$ / RUN / area

For θ in radians, insert / ▼ / R→D / between steps 04 and 05.

X	·	00
stop	0	01
X	·	02
(	6	03
stop	0	04
—	F	05
#	3	06
9	9	07
0	0	08
=	—	09
▼	A	10
D→R	3	11
cos	8	12
)	6	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

SPHERES

Surface area and volume

$A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

Execution:

radius / RUN / surface area / RUN / volume

X	·	00
(	6	01
X	·	02
X	·	03
#	3	04
1	1	05
2	2	06
·	A	07
5	5	08
6	6	09
6	6	10
3	3	11
7	7	12
1	1	13
=	—	14
stop	0	15
)	6	16
÷	G	17
#	3	18
3	3	19
=	—	20
stop	0	21
▼	A	22
goto	2	23
0	0	24
0	0	25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

SPHERES

Radius from volume

$r = \sqrt[3]{\frac{3V}{4\pi}}$

Execution:

V / RUN / r

X	·	00
#	3	01
·	A	02
2	2	03
3	3	04
8	8	05
7	7	06
3	3	07
2	2	08
4	4	09
=	—	10
ln	4	11
÷	G	12
#	3	13
3	3	14
=	—	15
▼	A	16
e^x	4	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
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		27
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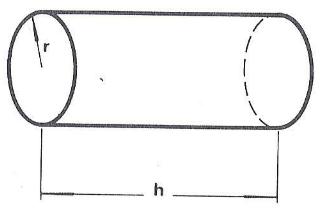
		34
		35

CYLINDERS



Volume = $\pi r^2 h$

Area of curved surface = $2\pi rh$

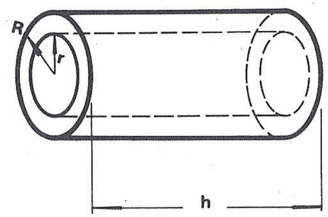
Total surface area = $2\pi r(r + h)$

Execution:

r / RUN / h / RUN / volume / RUN / area of curved surface / RUN / total surface area

sto	2	00
X	·	01
X	·	02
#	3	03
6	6	04
·	A	05
2	2	06
8	8	07
3	3	08
1	1	09
8	8	10
5	5	11
3	3	12
+	E	13
(	6	14
÷	G	15
#	3	16
2	2	17
X	·	18
stop	0	19
÷	G	20
stop	0	21
rcl	5	22
+	E	23
)	6	24
stop	0	25
=	—	26
stop	0	27
▼	A	28
goto	2	29
0	0	30
0	0	31
		32
		33
		34
		35

HOLLOW CYLINDRICAL TUBE



Area of curved surface = $2\pi h(R + r)$

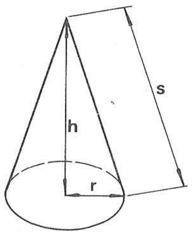
Volume = $\pi h(R^2 - r^2)$

Execution:

R / RUN / r / RUN / h / RUN / area of curved surface / RUN / volume

+	E	00
stop	0	01
sto	2	02
÷	G	03
#	3	04
2	2	05
—	F	06
▼	A	07
MEx	5	08
=	—	09
▼	A	10
MEx	5	11
X	·	12
stop	0	13
X	·	14
#	3	15
1	1	16
2	2	17
·	A	18
5	5	19
6	6	20
6	6	21
X	·	22
stop	0	23
rcl	5	24
=	—	25
stop	0	26
▼	A	27
goto	2	28
0	0	29
0	0	30
		31
		32
		33
		34
		35

RIGHT CIRCULAR CONE



Volume = $\frac{\pi r^2 h}{3}$

Curved surface area = $\pi r\sqrt{r^2 + h^2}$

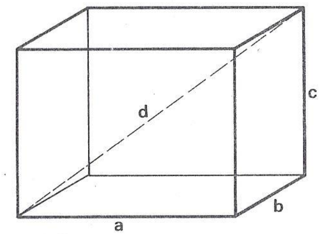
Total surface area = $\pi r\left(r + \sqrt{r^2 + h^2}\right)$

Execution:

h / RUN / r / RUN / area of curved surface / ▲▼ / rcl / area of base / RUN / total surface area / RUN / volume

X	·	00
(	6	01
÷	G	02
stop	0	03
sto	2	04
X	·	05
+	E	06
#	3	07
1	1	08
=	–	09
√X	1	10
▼	A	11
MEx	5	12
X	·	13
X	·	14
#	3	15
3	3	16
·	A	17
1	1	18
4	4	19
1	1	20
6	6	21
X	·	22
▼	A	23
MEx	5	24
+	E	25
stop	0	26
=	–	27
stop	0	28
rcl	5	29
)	6	30
÷	G	31
#	3	32
3	3	33
=	–	34
stop	0	35

RECTANGULAR PARALLELEPIPED



Diagonal:

$d = \sqrt{a^2 + b^2 + c^2}$

Execution:

a / RUN / b / RUN / c / RUN / d

X	·	00
+	E	01
(	6	02
stop	0	03
X	·	04
)	6	05
+	E	06
(	6	07
stop	0	08
X	·	09
)	6	10
=	–	11
√X	1	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
0	0	17
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RECTANGULAR PARALLELEPIPED

Surface area

sto	2	00
stop	0	01
+	E	02
(	6	03
X	·	04
rcl	5	05
=	—	06
▼	A	07
MEx	5	08
)	6	09
X	·	10
stop	0	11
+	E	12
rcl	5	13
+	E	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
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$A = 2(ab + ac + bc)$

Execution:

a / RUN / b / RUN / c / RUN / area

DISTANCE BETWEEN TWO POINTS IN SPACE

—	F	00
stop	0	01
X	·	02
+	E	03
(	6	04
stop	0	05
—	F	06
stop	0	07
X	·	08
)	6	09
+	E	10
(	6	11
stop	0	12
—	F	13
stop	0	14
X	·	15
)	6	16
=	—	17
$\sqrt{x}$	1	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
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$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

points are (x_1, y_1, z_1) and (x_2, y_2, z_2)

Execution:

x_1 / RUN / x_2 / RUN / y_1 / RUN / y_2 / RUN / z_1 / RUN / z_2 / RUN / d

COORDINATE CONVERSION

Polar to cartesian

θ in radians, $-\pi < \theta < \pi$, $\theta \neq 0$

Execution:

r / RUN / θ / RUN / x / RUN / y

If $\theta = 0$, x = r and y = 0

If $\theta = \pi$, x = –r and y = 0

×	·	00
(	6	01
stop	0	02
÷	G	03
#	3	04
2	2	05
=	—	06
tan	9	07
sto	2	08
÷	G	09
+	E	10
rcl	5	11
÷	G	12
+	E	13
)	6	14
=	—	15
▼	A	16
MEx	5	17
—	F	18
(	6	19
÷	G	20
)	6	21
÷	G	22
#	3	23
2	2	24
—	F	25
×	·	26
rcl	5	27
=	—	28
stop	0	29
rcl	5	30
stop	0	31
=	—	32
=	—	33
=	—	34
=	—	35

COORDINATE CONVERSION

Cartesian to polar

Restriction: $y \neq 0$

If y = 0, r = |x|

and $\theta = 0$ if $x \geqslant 0$

π if $x < 0$

Execution:

x / RUN / y / RUN / r / RUN / θ

÷	G	00
(	6	01
×	·	02
÷	G	03
stop	0	04
sto	2	05
+	E	06
rcl	5	07
×	·	08
rcl	5	09
=	—	10
$\sqrt{x}$	1	11
stop	0	12
)	6	13
+	E	14
#	3	15
1	1	16
÷	G	17
#	3	18
2	2	19
=	—	20
$\sqrt{x}$	1	21
▼	A	22
arccos	8	23
+	E	24
×	·	25
(	6	26
rcl	5	27
×	·	28
÷	G	29
$\sqrt{x}$	1	30
rcl	5	31
)	6	32
=	—	33
stop	0	34
=	—	35

RADIUS OF CURVATURE

$$r = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

Execution:

$/ \frac{dy}{dx}$ / RUN / $\frac{d^2y}{dx^2}$ / RUN / r

X	·	00
+	E	01
#	3	02
1	1	03
X	·	04
(	6	05
$\sqrt{x}$	1	06
)	6	07
÷	G	08
stop	0	09
=	—	10
stop	0	11
▼	A	12
goto	2	13
0	0	14
0	0	15
		16
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HAVERSINE AND INVERSE HAVERSINE, VERSINE AND SUVERSINE

Haversine:

pre-execution: ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

$\theta°$ / RUN / hav θ

/ + / = / vers θ / – / + / 2 / = / suvers θ

Inverse haversine:

pre-execution: ▲▼ / ▲▼ / goto / 1 / 4 /

Execution:

hav θ / RUN / $\theta°$

vers θ / ÷ / 2 / = / RUN / $\theta°$

suvers θ / – / + / 2 / ÷ / 2 / = / RUN / $\theta°$

Range $0 \leqslant \theta° \leqslant 180$

For vers θ see post and pre-execution.

▼	A	00
D→R	3	01
÷	G	02
#	3	03
2	2	04
=	—	05
sin	7	06
X	·	07
=	—	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
$\sqrt{x}$	1	14
▼	A	15
arcsin	7	16
+	E	17
=	—	18
▼	A	19
R→D	6	20
stop	0	21
▼	A	22
goto	2	23
1	1	24
4	4	25
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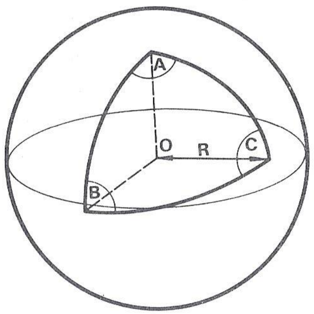
		34
		35

AREA OF A SPHERICAL TRIANGLE



Area = $(A + B + C - \pi)R^2$

A, B, C in degrees

Execution:

A / RUN / B / RUN / C / RUN / R / RUN / area

+	E	00
stop	0	01
+	E	02
stop	0	03
−	F	04
#	3	05
1	1	06
8	8	07
0	0	08
=	−	09
▼	A	10
D→R	3	11
X	·	12
(	6	13
stop	0	14
X	·	15
)	6	16
=	−	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
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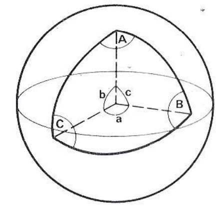
SPHERICAL TRIANGLES: SINE RULE

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$



Angles in degrees

Execution:

a / RUN / A / RUN / B / RUN / b / C / RUN / c

or

A / RUN / a / RUN / b / RUN / B / c / RUN / C

Note: If a result of 0 appears, the final arcsin had an out-of-range argument and the result is impossible for the particular angles given, or else very close to 90°.

For angle A > 90°, compute using 180 / − / A / = / etc.

Special execution: navigation

To find course from place 2 to place 1

$$\sin C = \frac{\sin (E_1 - E_2) \cos N_2}{\sin d}$$

Execution:

E_1 / − / E_2 / RUN / d / RUN / 90 / − / N_2 / = / RUN / C

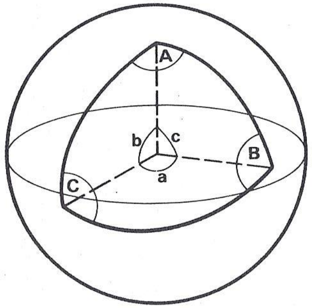
where E_1 = easterly longitude of place 1
E_2 = easterly longitude of place 2
N_2 = north latitude of place 2
d = angular distance between places 1 and 2

(For westerly longitudes or south latitudes, change sign of angle.)

▼	A	00
D→R	3	01
sin	7	02
÷	G	03
(	6	04
stop	0	05
▼	A	06
D→R	3	07
sin	7	08
)	6	09
X	·	10
sto	2	11
(	6	12
stop	0	13
▼	A	14
D→R	3	15
sin	7	16
)	6	17
=	−	18
▼	A	19
arcsin	7	20
▼	A	21
R→D	6	22
stop	0	23
▼	A	24
D→R	3	25
sin	7	26
X	·	27
rcl	5	28
▼	A	29
goto	2	30
1	1	31
8	8	32
		33
		34
		35

SPHERICAL TRIANGLES:

Cosine Rule



cos a = cos b (cos c + sin c tan b cos A)

Execution:

c / RUN / A / RUN / b / RUN / b / RUN / a

Navigation

To find great circle distance between places 1 and 2

1. Latitude N_1 longitude E_1 (–ve if W)
2. Latitude N_2 longitude E_2 (–ve if W)

Execution:

90 / – / N_2 / = / RUN / E_1 / – / E_2 / RUN / 90 / – / N_1 / RUN / 90 / – / N_1 / RUN / d (degrees)

X / 111·19 / = / distance in km

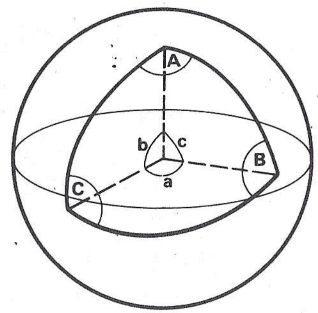
or X / 69·41 / = / distance in miles

For angles greater than 90° use appropriate reductions to first quadrant.

▼	A	00
D→R	3	01
sto	2	02
sin	7	03
X	·	04
(	6	05
stop	0	06
▼	A	07
D→R	3	08
cos	8	09
)	6	10
X	·	11
(	6	12
stop	0	13
▼	A	14
D→R	3	15
tan	9	16
)	6	17
+	E	18
(	6	19
rcl	5	20
cos	8	21
)	6	22
X	·	23
(	6	24
stop	0	25
▼	A	26
D→R	3	27
cos	8	28
)	6	29
=	–	30
▼	A	31
arccos	8	32
▼	A	33
R→D	6	34
stop	0	35

SPHERICAL TRIANGLES

The Cosine Rule – to find an angle or side given three sides or angles



$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$\cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C}$$

Execution:

Angles in radians

c / RUN / b / RUN / – / RUN / a / RUN / A

C / RUN / B / RUN / RUN / A / RUN / a

For angles in degrees use

▲▼ / ▲▼ / D→R / after each angle.

sto	2	00
sin	7	01
▼	A	02
MEx	5	03
cos	8	04
X	·	05
(	6	06
stop	0	07
sin	7	08
X	·	09
▼	A	10
MEx	5	11
=	–	12
▼	A	13
MEx	5	14
X	·	15
–	F	16
#	3	17
1	1	18
–	F	19
)	6	20
stop	0	21
+	E	22
(	6	23
stop	0	24
cos	8	25
)	6	26
÷	G	27
rcl	5	28
=	–	29
▼	A	30
arccos	8	31
stop	0	32
=	–	33
=	–	34
=	–	35

SPHERICAL TRIANGLES:

Half-angle tangent formula

$$\tan \frac{A}{2} = \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin (s - a) \sin s}}$$

$$\text{where } s = \frac{a + b + c}{2}$$

$$\tan \frac{a}{2} = \sqrt{\frac{\cos (\pi - S) \cos (S - A)}{\cos (S - B) \cos (S - C)}}$$

$$\text{where } S = \frac{A + B + C}{2}$$

Execution:

Angles in radians

a / + / b / + / c / RUN / b / RUN / a / RUN /

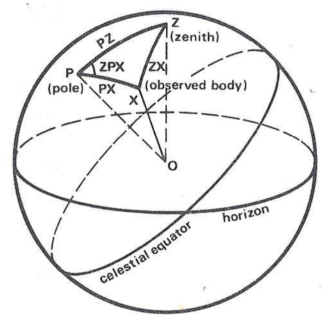
$\frac{A}{2}$ / = / A

For the cosine version change all / sin / steps to / cos /.

sto	2	00
÷	G	01
#	3	02
2	2	03
—	F	04
▼	A	05
MEx	5	06
=	—	07
sin	7	08
÷	G	09
(	6	10
rcl	5	11
sin	7	12
)	6	13
X	·	14
(	6	15
stop	0	16
—	F	17
rcl	5	18
=	—	19
sin	7	20
)	6	21
÷	G	22
(	6	23
stop	0	24
—	F	25
rcl	5	26
=	—	27
sin	7	28
)	6	29
=	—	30
√x	1	31
▼	A	32
arctan	9	33
+	E	34
stop	0	35

SPHERICAL TRIANGLES

Solving the PZX triangle



hav ZX = hav (PX ~ PZ) + sin PX sin PZ hav ∠ZPX

= hav (L ~ D) + cos L cos D hav ∠ZPX

(for the second formula use cos at steps 10 and 27 instead of sin)

ZX is the calculated zenith distance (CZD)

Enter south latitudes as —ve

Execution:

Angles in radians —

∠ZPX / RUN / PX / RUN / PZ / RUN / + / = / ZX

Angles in degrees —

∠ZPX° / ▲▼ / ▲▼ / D→R / RUN / PX° / ▲▼ / ▲▼ / D→R / RUN / PZ° / ▲▼ / ▲▼ / D→R / RUN / + / = / ▲▼ / ▲▼ / R→D / ZX°

Intercept I = CZD — TZD (calculated — true zenith distance)

Post-execution:

/ — / TZD / X / 60 / = / I′

(I in minutes of arc or miles approx.)

÷	G	00
+	E	01
÷	G	02
=	—	03
sin	7	04
X	·	05
X	·	06
(	6	07
stop	0	08
sto	2	09
sin	7	10
)	6	11
X	·	12
(	6	13
stop	0	14
—	F	15
▼	A	16
MEx	5	17
÷	G	18
#	3	19
2	2	20
=	—	21
sin	7	22
X	·	23
=	—	24
▼	A	25
MEx	5	26
sin	7	27
)	6	28
+	E	29
rcl	5	30
=	—	31
√x	1	32
▼	A	33
arcsin	7	34
stop	0	35

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3 Physics & Engineering

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Properties of Matter
Fluids
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Physics & Engineering

3

CONTENTS

How to use these programs

Each program is arranged as follows:

1. On the left of the page, explanatory information and the 'execution sequence', the sequence of keystrokes necessary for running the program. Results displayed are printed in gold.
2. In the first column on the right hand side of the page, the sequence of keystrokes which make up the program.
3. In the second and third columns on the right hand side of the page, the program in check symbol and step number form (see section on checking the program).

Notes

1. Where a key has more than one function, the relevant function is printed as the keystroke in the first column
   e.g. the keystroke [8] (cos / arccos) may appear as 8, cos or arccos.
2. The symbol ▼ within a program always refers to the key [·/EE/–] (▼)
3. The symbol # refers to [3] (ChN/#)
4. The abbreviation gin is 'go if neg' and so refers to the key [1] (go if neg)

Entering the program

To enter a program into the calculator:

1. Press [▲▼] [▲▼] [2] (go to) [0] [0]	Display shows step programmed at 00 in check symbol form as described below.
2. Press [▲▼] [RUN] (learn)	No change in display.
3. Press the sequence of keys for the program as shown in the first column of the program page.	At each stage the step about to be overwritten is displayed. When the machine is first switched on every step is zero.
4. Press [C/CE]	Normal number display is resumed.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]	The step programmed at 00 will be displayed.

Checking the program

Each of the programs in the library is shown in check symbol form in the second column on the right-hand side of the page.

Press [▲▼] [C/CE] (step) repeatedly, and at each stage the check symbol will appear on the left of the display with the step number on the right. Ignore the four zeros in the display.

e.g. A.0000 03 (check symbol, step number)

After stepping through the program, press

[▲▼] [▲▼] [2] (go to) [0] [0] before execution.

Finally, press [C/CE] and the program is ready for use.

Correcting the program

If the check symbol for a particular step number is not as indicated in the last two columns of the program page:

1. Press [▲▼] [▲▼] [2] (go to)
   followed by the step number if the appropriate step number is not already displayed.
2. Press [▲▼] [RUN] (learn)
3. Enter the correct keystroke. The display will then show the next step in the program. If this is also incorrect, enter the correct keystroke. At each stage, the step about to be overwritten will be displayed.
4. When correction has been completed, press [C/CE]. Any step which has not been overwritten will not be affected.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]

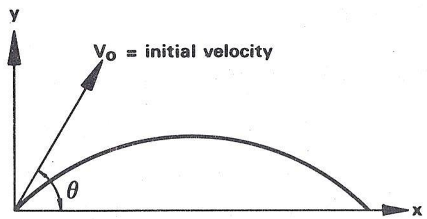
Note
To restore normal use of the calculator after entering or checking the program, press [C/CE]

Running the program

Press the sequence of keys as shown in the program library in the execution sequence. Results displayed are printed in gold.

PROJECTILES

Position relative to point of projection after time t



$x = v_o t \cos\theta$

$y = v_o t \sin\theta - \frac{gt^2}{2}$

Execution:

θ° / RUN / v_o / RUN / t / RUN / x / RUN / y

In S.I. units; g taken as 9·81ms⁻².

▼	A	00
D→R	3	01
sto	2	02
tan	9	03
X	·	04
(	6	05
rcl	5	06
cos	8	07
X	·	08
stop	0	09
X	·	10
stop	0	11
sto	2	12
)	6	13
stop	0	14
—	F	15
(	6	16
rcl	5	17
X	·	18
X	·	19
#	3	20
4	4	21
·	A	22
9	9	23
0	0	24
5	5	25
=	—	26
)	6	27
=	—	28
stop	0	29
▼	A	30
goto	2	31
0	0	32
0	0	33
		34
		35

PROJECTILES

Range, maximum height and time of flight

$$T = \frac{2v_o}{g} \sin\theta$$

$$R = \frac{2v_o^2}{g} \sin\theta \cos\theta$$

$$H = \frac{v_o^2}{2g} \sin^2\theta$$

Execution:

θ° / RUN / v_o / RUN / time of flight / RUN / maximum height / RUN / range

In S.I. units; g taken as 9·81ms⁻².

▼	A	00
D→R	3	01
sto	2	02
sin	7	03
X	·	04
stop	0	05
X	·	06
#	3	07
·	A	08
2	2	09
0	0	10
4	4	11
X	·	12
stop	0	13
X	·	14
#	3	15
1	1	16
·	A	17
2	2	18
2	2	19
6	6	20
÷	G	21
stop	0	22
(	6	23
rcl	5	24
tan	9	25
)	6	26
+	E	27
+	E	28
=	—	29
stop	0	30
▼	A	31
goto	2	32
0	0	33
0	0	34
		35

PROJECTILES

Necessary angle of projection for given range with given speed of projection

$\sin 2\alpha = \frac{Rg}{v^2}$ giving two possible angles α_1 and α_2.

Execution:

v / RUN / R / RUN / α_1° / RUN / α_2°

In S.I. units; g taken as $9{\cdot}81 ms^{-2}$.

X	·	00
÷	G	01
X	·	02
#	3	03
9	9	04
·	A	05
8	8	06
1	1	07
X	·	08
stop	0	09
=	—	10
▼	A	11
arcsin	7	12
▼	A	13
R→D	3	14
÷	G	15
#	3	16
2	2	17
—	F	18
stop	0	19
#	3	20
9	9	21
0	0	22
—	F	23
=	—	24
stop	0	25
▼	A	26
goto	2	27
0	0	28
0	0	29
		30
		31
		32
		33
		34
		35

PARALLELOGRAM LAW FOR FORCES

$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$

Execution:

P / RUN / Q / RUN / α° / RUN / R

Range: $0 \leqslant \alpha^\circ \leqslant 180^\circ$

E may appear if α is close to 0° or 180°

sto	2	00
stop	0	01
X	·	02
(	6	03
+	E	04
rcl	5	05
=	—	06
▼	A	07
MEx	5	08
)	6	09
X	·	10
(	6	11
stop	0	12
—	F	13
#	3	14
9	9	15
0	0	16
—	F	17
=	—	18
▼	A	19
D→R	3	20
sin	7	21
—	F	22
#	3	23
1	1	24
+	E	25
)	6	26
+	E	27
(	6	28
rcl	5	29
X	·	30
)	6	31
=	—	32
$\sqrt{x}$	1	33
stop	0	34
=	—	35

CONSTANT ACCELERATION MOTION

X	·	00
(	6	01
X	·	02
stop	0	03
÷	G	04
#	3	05
2	2	06
+	E	07
sto	2	08
+	E	09
stop	0	10
–	F	11
stop	0	12
rcl	5	13
)	6	14
=	–	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
		26
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u = initial velocity
v = final velocity
s = distance covered
f = acceleration
t = time

$v = u + ft$

$s = ut + \frac{ft^2}{2}$

Execution:

t / RUN / f / RUN / u / RUN / v /RUN / s

CONSTANT ACCELERATION MOTION

X	·	00
+	E	01
(	6	02
stop	0	03
X	·	04
stop	0	05
+	E	06
)	6	07
=	–	08
$\sqrt{x}$	1	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
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For notation see page 12

$v = \sqrt{u^2 + 2fs}$

Execution:

u / RUN / f / RUN / s / RUN / v

This gives the absolute value of v; other considerations must be used to determine the correct sign.

CONSTANT ACCELERATION MOTION

For notation see page 12

(i) $t = \frac{v - u}{f}$

$s = \frac{v^2 - u^2}{2f}$

Execution:

v / RUN / u / RUN / f / RUN / t / RUN / s

(ii) $f = \frac{v - u}{t}$

Execution:

v / RUN / u / RUN / t / RUN / f / RUN

(iii) $f = \frac{v^2 - u^2}{2s}$

Execution:

v / RUN / u / RUN / s / RUN / RUN / f

—	F	00
stop	0	01
sto	2	02
÷	G	03
(	6	04
÷	G	05
#	3	06
2	2	07
+	E	08
rcl	5	09
=	—	10
sto	2	11
stop	0	12
)	6	13
X	·	14
stop	0	15
rcl	5	16
=	—	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
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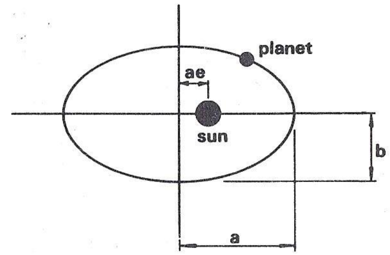
PLANETARY MOTION



Kepler's law:
orbit is an ellipse with sun at one focus.

$$r = \frac{p}{1 + e\cos\theta} = \frac{b\sqrt{1 - e^2}}{1 + e\cos\theta} = \frac{b^2}{a(1 + e\cos\theta)};$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

Execution:

$\theta°$ / RUN / e / RUN / p / RUN / r

X	·	00
=	—	01
$\sqrt{x}$	1	02
—	F	03
#	3	04
9	9	05
0	0	06
—	F	07
=	—	08
▼	A	09
D→R	3	10
sin	7	11
X	·	12
stop	0	13
+	E	14
#	3	15
1	1	16
÷	G	17
X	·	18
stop	0	19
=	—	20
stop	0	21
▼	A	22
goto	2	23
0	0	24
0	0	25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

PLANETARY MOTION

×	·	00
=	–	01
$\sqrt{x}$	1	02
–	F	03
#	3	04
9	9	05
0	0	06
–	F	07
=	–	08
▼	A	09
D→R	3	10
sin	7	11
×	·	12
stop	0	13
sto	2	14
+	E	15
#	3	16
1	1	17
÷	G	18
×	·	19
(	6	20
rcl	5	21
×	·	22
–	F	23
+	E	24
#	3	25
1	1	26
=	–	27
$\sqrt{x}$	1	28
)	6	29
×	·	30
stop	0	31
=	–	32
stop	0	33
=	–	34
=	–	35

For notation see page 15

Execution:

$\theta°$ / RUN / e / RUN / b / RUN / r

PLANETARY MOTION

sto	2	00
÷	G	01
stop	0	02
×	·	03
▼	A	04
MEx	5	05
=	–	06
▼	A	07
MEx	5	08
▼	A	09
arcsin	7	10
cos	8	11
×	·	12
(	6	13
stop	0	14
×	·	15
=	–	16
$\sqrt{x}$	1	17
–	F	18
#	3	19
9	9	20
0	0	21
–	F	22
=	–	23
▼	A	24
D→R	3	25
sin	7	26
)	6	27
+	E	28
#	3	29
1	1	30
÷	G	31
rcl	5	32
÷	G	33
=	–	34
stop	0	35

For notation see page 15

Execution:

b / RUN / a / RUN / $\theta°$ / RUN / r

DOPPLER EFFECT
(non-relativistic)

For sound waves, etc.

v_o = observer velocity
v_s = source velocity
f_s = transmitted frequency
f_o = observed frequency
c = velocity of wave

Given observed frequency, to find transmitted frequency.

$$f_s = \left(\frac{c + v_o}{c - v_s}\right) f_o$$

Execution:

c / RUN / v_o / RUN / v_s / RUN / f_o / RUN / f_s

sto	2	00
+	E	01
stop	0	02
÷	G	03
(	6	04
rcl	5	05
—	F	06
stop	0	07
)	6	08
X	·	09
stop	0	10
=	—	11
stop	0	12
▼	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
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DOPPLER EFFECT
(non-relativistic)

For notation see page 18

Given transmitted frequency, to find observed frequency.

Execution:

c / RUN / v_s / RUN / v_o / RUN / f_s / RUN / f_o

sto	2	00
—	F	01
stop	0	02
÷	G	03
(	6	04
stop	0	05
+	E	06
rcl	5	07
)	6	08
X	·	09
stop	0	10
=	—	11
stop	0	12
▼	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
		20
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		31
		32
		33
		34
		35

DOPPLER EFFECT
(non-relativistic)

—	F	00
(	6	01
+	E	02
stop	0	03
X	·	04
stop	0	05
÷	G	06
stop	0	07
)	6	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
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		22
		23
		24
		25
		26
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For notation see page 18

Given both frequencies, to find source velocity.

Execution:

c / RUN / v_o / RUN / f_o / RUN / f_s / RUN / v_s

DOPPLER EFFECT
(relativistic)

(Red shift or blue shift)

÷	G	00
stop	0	01
X	·	02
sto	2	03
(	6	04
stop	0	05
X	·	06
=	—	07
$\sqrt{x}$	1	08
—	F	09
#	3	10
9	9	11
0	0	12
=	—	13
▼	A	14
D→R	3	15
sin	7	16
)	6	17
+	E	18
#	3	19
1	1	20
÷	G	21
X	·	22
(	6	23
rcl	5	24
▼	A	25
arcsin	7	26
cos	8	27
)	6	28
=	—	29
sto	2	30
stop	0	31
rcl	5	32
=	—	33
stop	0	34
=	—	35

f_s = source frequency
f_o = observed frequency
c = speed of light = $2{\cdot}997925 \times 10^8 ms^{-1}$
v = speed of source
θ = direction of motion of source relative to observer

$$f_o = f_s \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}\cos\theta}$$

Execution:

(i) v / RUN / c / RUN / θ / RUN / f_s / X / RUN / f_o

(ii) v / RUN / c / RUN / θ / RUN / f_o / ÷ / RUN / f_s

DOPPLER EFFECT (relativistic)

(Red shift or blue shift)

÷	G	00
stop	0	01
—	F	02
#	3	03
1	1	04
÷	G	05
(	6	06
+	E	07
#	3	08
2	2	09
—	F	10
)	6	11
=	—	12
$\sqrt{x}$	1	13
sto	2	14
stop	0	15
rcl	5	16
=	—	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
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		30
		31
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		34
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Source receding from observer at velocity v.
Source frequency f_s, observed frequency f_o.

$$f_o = f_s \frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}} = f_s \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

Given v and one frequency, to find the other frequency.

Execution:

(i) v / RUN / c / RUN / f_s / X / RUN / f_o

(ii) v / RUN / c / RUN / f_o / ÷ / RUN / f_s

DOPPLER EFFECT (relativistic)

(Red shift or blue shift)

÷	G	00
stop	0	01
X	·	02
—	F	03
#	3	04
1	1	05
÷	G	06
(	6	07
+	E	08
#	3	09
2	2	10
—	F	11
)	6	12
X	·	13
stop	0	14
=	—	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
		21
		22
		23
		24
		25
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		28
		29
		30
		31
		32
		33
		34
		35

To find v, given f_o and f_s.

Execution:

f_o / RUN / f_s / RUN / c / RUN / v

If the wavelengths λ_o and λ_s are known:

Execution:

λ_s / RUN / λ_o / RUN / c / RUN / v

If v is negative, motion is towards observer.

RELATIVITY

Fitzgerald contraction, time dilation and mass change.

÷	G	00
stop	0	01
X	·	02
—	F	03
+	E	04
#	3	05
1	1	06
=	—	07
$\sqrt{x}$	1	08
sto	2	09
stop	0	10
rcl	5	11
=	—	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
0	0	17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
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		33
		34
		35

$$T' = T\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

$$L' = L\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

$$M' = M\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Execution:

(i) v / RUN / c / RUN / T / X / RUN / T′

(ii) v / RUN / c / RUN / L / X / RUN / L′

(iii) v / RUN / c / RUN / M / ÷ / RUN / M′

LORENTZ TRANSFORMATION

▼	A	00
arcsin	7	01
sto	2	02
cos	8	03
÷	G	04
X	·	05
stop	0	06
—	F	07
(	6	08
rcl	5	09
tan	9	10
X	·	11
stop	0	12
)	6	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

(i) $$X' = \frac{X - \beta c T}{\sqrt{1 - \beta^2}}$$

(ii) $$T' = \frac{T - \frac{\beta X}{c}}{\sqrt{1 - \beta^2}}$$

where $\beta = \frac{v}{c}$

(a) Units such that c = 1

Execution:

(i) β / RUN / X / RUN / T / RUN / X′

(ii) β / RUN / T / RUN / X / RUN / T′

(b) Any consistent units

Execution:

(i) v / ÷ / c / = / RUN / X / RUN / T / X / c / RUN / X′

(ii) v / ÷ / c / = / RUN / T / RUN / X / ÷ / c / RUN / T′

COMPOUND PENDULUM

T = period
k_o = radius of gyration about pivot
k_g = radius of gyration about c.g.
r = distance from pivot to c.g.
l = length of simple equivalent pendulum

$$T = \frac{2\pi k_o}{\sqrt{gr}} \qquad l = \frac{k_o^2}{r}$$

Execution:

r / RUN / k_o / RUN / T / RUN / l

In S.I. units; g taken as $9{\cdot}81ms^{-2}$.

√x	1	00
÷	G	01
X	·	02
stop	0	03
X	·	04
sto	2	05
#	3	06
2	2	07
·	A	08
0	0	09
0	0	10
6	6	11
1	1	12
=	–	13
stop	0	14
rcl	5	15
X	·	16
=	–	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

COMPOUND PENDULUM

Notation as on page 26

Use $k_o = \sqrt{k_g^2 + r^2}$

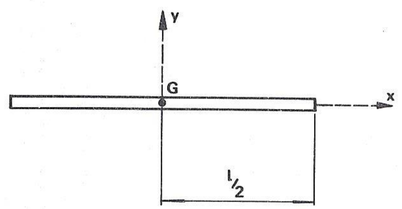
Execution:

r / RUN / k_g / RUN / T / RUN / l

sto	2	00
X	·	01
+	E	02
(	6	03
stop	0	04
X	·	05
)	6	06
÷	G	07
rcl	5	08
=	–	09
sto	2	10
√x	1	11
X	·	12
#	3	13
2	2	14
·	A	15
0	0	16
0	0	17
6	6	18
1	1	19
=	–	20
stop	0	21
rcl	5	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
0	0	27
		28
		29
		30
		31
		32
		33
		34
		35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Straight Rod



$k^2_{xx} = 0$

$k^2_{yy} = \frac{l^2}{12}$

Execution:

l / RUN / k^2_{yy}

Notation throughout this section:

G = position of centre of gravity

k_{xx} = radius of gyration about x-axis through G

k_{yy} = radius of gyration about y-axis through G

X	·	00
÷	G	01
#	3	02
1	1	03
2	2	04
=	–	05
stop	0	06
▼	A	07
goto	2	08
0	0	09
0	0	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
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		34
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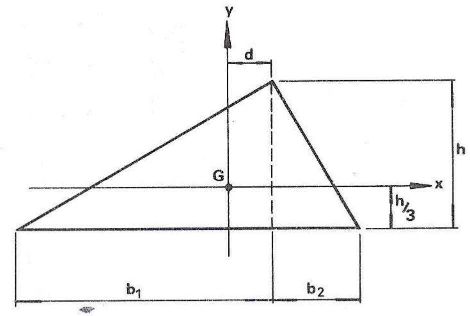
CENTRE OF GRAVITY AND RADIUS OF GYRATION

Triangular Lamina



For notation see page 28

$d = \frac{b_1 - b_2}{3}$

$k^2_{xx} = \frac{h^2}{18}$

$k^2_{yy} = \frac{b_1^2 + b_1 b_2 + b_2^2}{18}$

Execution:

b_1 / RUN / b_2 / RUN / d / RUN / k^2_{yy} / h / RUN / k^2_{xx}

X	·	00
(	6	01
–	F	02
stop	0	03
sto	2	04
÷	G	05
#	3	06
3	3	07
X	·	08
stop	0	09
÷	G	10
#	3	11
2	2	12
=	–	13
▼	A	14
MEx	5	15
)	6	16
÷	G	17
#	3	18
6	6	19
+	E	20
rcl	5	21
=	–	22
stop	0	23
X	·	24
÷	G	25
#	3	26
1	1	27
8	8	28
=	–	29
stop	0	30
▼	A	31
goto	2	32
0	0	33
0	0	34
		35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Rectangular lamina

X	·	00
÷	G	01
#	3	02
1	1	03
2	2	04
=	—	05
stop	0	06
▼	A	07
goto	2	08
0	0	09
0	0	10
		11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
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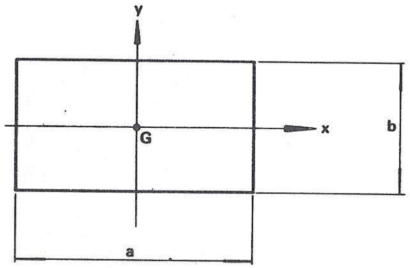
		30
		31
		32
		33
		34
		35



For notation see page 28

$$k_{xx}^2 = \frac{b^2}{12}$$

$$k_{yy}^2 = \frac{a^2}{12}$$

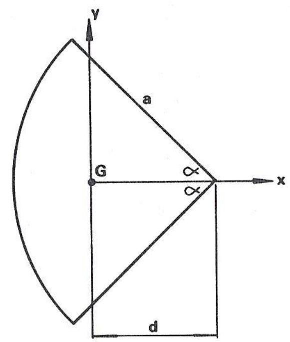
Execution:

b / RUN / k_{xx}^2 / a / RUN / k_{yy}^2

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Sector of circular lamina

sto	2	00
sin	7	01
÷	G	02
rcl	5	03
=	—	04
▼	A	05
MEx	5	06
cos	8	07
X	·	08
(	6	09
stop	0	10
÷	G	11
#	3	12
3	3	13
+	E	14
X	·	15
▼	A	16
MEx	5	17
=	—	18
stop	0	19
)	6	20
—	F	21
rcl	5	22
X	·	23
rcl	5	24
—	F	25
X	·	26
#	3	27
9	9	28
÷	G	29
#	3	30
1	1	31
6	6	32
=	—	33
stop	0	34
=	—	35



For notation see page 28

$$d = \frac{2a \sin \alpha}{3\alpha}$$

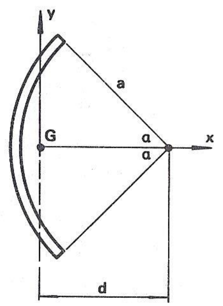
$$k_{xx}^2 = \frac{a^2}{4}\left(1 - \frac{\sin 2\alpha}{2\alpha}\right)$$

Execution:

α (in radians) / RUN / a / RUN / d
RUN / k_{xx}^2

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Curved rod (arc of a circle)



For notation see page 28

$$d = a\frac{\sin\alpha}{\alpha}$$

$$k_{xx}^2 = \frac{a^2}{2}\left(1 - \frac{\sin 2\alpha}{2\alpha}\right)$$

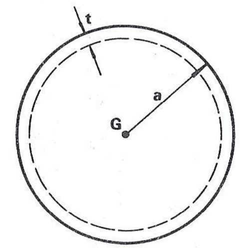
Execution:

$\alpha°$ / RUN / a / RUN / d / RUN / k_{xx}^2

▼	A	00
D→R	3	01
sto	2	02
sin	7	03
÷	G	04
rcl	5	05
=	—	06
▼	A	07
MEx	5	08
cos	8	09
X	·	10
(	6	11
stop	0	12
X	·	13
▼	A	14
MEx	5	15
)	6	16
stop	0	17
—	F	18
rcl	5	19
X	·	20
rcl	5	21
—	F	22
÷	G	23
#	3	24
2	2	25
=	—	26
stop	0	27
▼	A	28
goto	2	29
0	0	30
0	0	31
		32
		33
		34
		35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Spherical shell



For notation see page 28

a = radius
t = thickness

Volume = $4\pi a^2 t$

$$k_{xx}^2 = k_{yy}^2 = k_{zz}^2 = \frac{2a^2}{3}$$

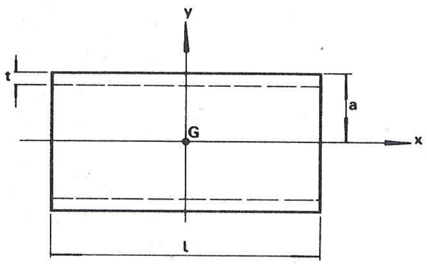
Execution:

a / RUN / k_{xx}^2 / t / RUN / volume

X	·	00
÷	G	01
#	3	02
1	1	03
·	A	04
5	5	05
X	·	06
stop	0	07
X	·	08
#	3	09
1	1	10
0	0	11
8	8	12
0	0	13
=	—	14
▼	A	15
D→R	3	16
stop	0	17
▼	A	18
goto	2	19
0	0	20
0	0	21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Thin-walled tube



For notation see page 28

a = radius
l = length
t = thickness

Volume = $2\pi alt$

$k_{xx}^2 = a^2$

$k_{yy}^2 = k_{zz}^2 = \frac{a^2}{2} + \frac{l^2}{12}$

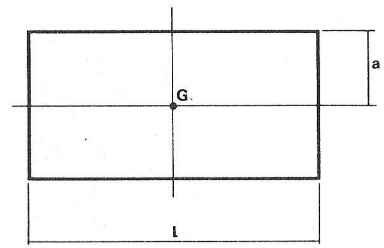
Execution:

l / RUN / a / RUN / t / RUN / volume / RUN / k_{xx}^2 / RUN / k_{yy}^2

X	·	00
+	E	01
(	6	02
√x	1	03
X	·	04
stop	0	05
sto	2	06
X	·	07
stop	0	08
X	·	09
#	3	10
3	3	11
6	6	12
0	0	13
=	—	14
▼	A	15
D→R	3	16
stop	0	17
rcl	5	18
X	·	19
X	·	20
stop	0	21
#	3	22
6	6	23
=	—	24
)	6	25
÷	G	26
#	3	27
1	1	28
2	2	29
=	—	30
stop	0	31
=	—	32
=	—	33
=	—	34
=	—	35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Solid cylinder



For notation see page 28

a = radius
l = length

Volume = $\pi a^2 l$

$k_{xx}^2 = \frac{a^2}{2}$

$k_{yy}^2 = \frac{a^2}{4} + \frac{l^2}{12} = k_{zz}^2$

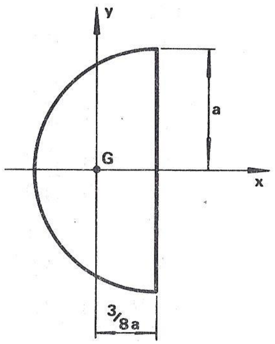
Execution:

a / RUN / k_{xx}^2 / l / RUN / volume / RUN / k_{yy}^2

X	·	00
÷	G	01
#	3	02
2	2	03
+	E	04
(	6	05
X	·	06
stop	0	07
sto	2	08
X	·	09
#	3	10
3	3	11
6	6	12
0	0	13
=	—	14
▼	A	15
D→R	3	16
stop	0	17
rcl	5	18
X	·	19
÷	G	20
#	3	21
6	6	22
=	—	23
)	6	24
÷	G	25
#	3	26
2	2	27
=	—	28
stop	0	29
▼	A	30
goto	2	31
0	0	32
0	0	33
		34
		35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Solid hemisphere



For notation see page 28

Volume = $\frac{2\pi a^3}{3}$

$k_{xx}^2 = \frac{2a^2}{5}$

$k_{yy}^2 = \frac{83a^2}{320}$

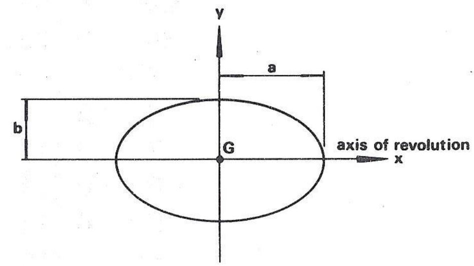
Execution:

a / RUN / volume / RUN / k_{xx}^2 / RUN / k_{yy}^2

sto	2	00
X	·	01
X	·	02
(	6	03
X	·	04
rcl	5	05
X	·	06
#	3	07
1	1	08
2	2	09
0	0	10
=	–	11
▼	A	12
D→R	3	13
stop	0	14
#	3	15
·	A	16
4	4	17
=	–	18
)	6	19
X	·	20
stop	0	21
#	3	22
8	8	23
3	3	24
÷	G	25
#	3	26
1	1	27
2	2	28
8	8	29
=	–	30
stop	0	31
=	–	32
=	–	33
=	–	34
=	–	35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Solid spheroid



For notation see page 28

(For sphere, a = b)

Volume = $\frac{4\pi ab^2}{3}$

$k_{xx}^2 = \frac{2b^2}{5}$

$k_{yy}^2 = \frac{a^2 + b^2}{5}$

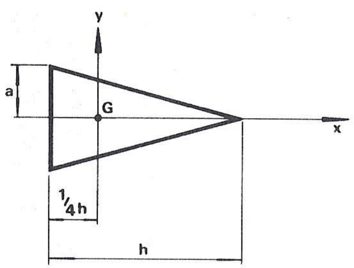
Execution:

b / RUN / k_{xx}^2 / a / RUN / volume / RUN / k_{yy}^2

X	·	00
X	·	01
#	3	02
·	A	03
4	4	04
+	E	05
(	6	06
X	·	07
stop	0	08
sto	2	09
X	·	10
#	3	11
6	6	12
0	0	13
0	0	14
=	–	15
▼	A	16
D→R	3	17
stop	0	18
rcl	5	19
X	·	20
X	·	21
#	3	22
·	A	23
4	4	24
=	–	25
)	6	26
÷	G	27
#	3	28
2	2	29
=	–	30
stop	0	31
=	–	32
=	–	33
=	–	34
=	–	35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Solid cone



For notation see page 28

h = height
a = radius of base

$$\text{Volume} = \frac{\pi a^2 h}{3}$$

$$k^2_{xx} = \frac{3a^2}{10}$$

$$k^2_{yy} = \frac{3(4a^2 + h^2)}{80}$$

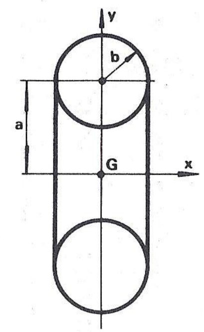
Execution:

a / RUN / k^2_{xx} / h / RUN / volume / RUN / k^2_{yy}

X	·	00
X	·	01
#	3	02
·	A	03
3	3	04
+	E	05
(	6	06
X	·	07
stop	0	08
sto	2	09
X	·	10
#	3	11
2	2	12
0	0	13
0	0	14
=	–	15
▼	A	16
D→R	3	17
stop	0	18
rcl	5	19
X	·	20
#	3	21
·	A	22
0	0	23
7	7	24
5	5	25
=	–	26
)	6	27
÷	G	28
#	3	29
2	2	30
=	–	31
stop	0	32
=	–	33
=	–	34
=	–	35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Toroid (circular section)



For notation see page 28

$$\text{Volume} = 2\pi^2 ab^2$$

$$k^2_{xx} = a^2 + \frac{3b^2}{4}$$

$$k^2_{yy} = \frac{a^2}{2} + \frac{5b^2}{8}$$

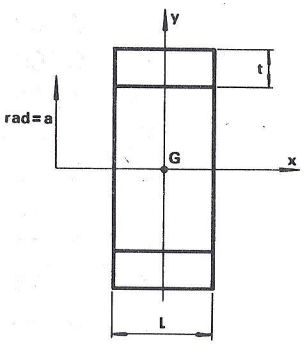
Execution:

b / RUN / a / RUN / volume / RUN / k^2_{xx} / RUN / k^2_{yy}

X	·	00
÷	G	01
#	3	02
4	4	03
=	–	04
sto	2	05
stop	0	06
X	·	07
+	E	08
(	6	09
$\sqrt{x}$	1	10
X	·	11
rcl	5	12
X	·	13
#	3	14
7	7	15
8	8	16
·	A	17
9	9	18
5	5	19
7	7	20
=	–	21
stop	0	22
#	3	23
3	3	24
X	·	25
rcl	5	26
)	6	27
÷	G	28
stop	0	29
#	3	30
2	2	31
+	E	32
rcl	5	33
=	–	34
stop	0	35

CENTRE OF GRAVITY AND RADIUS OF GYRATION

Toroid (rectangular section)



For notation see page 28

Volume = $2\pi atl$

$$k_{xx}^2 = a^2 + \frac{t^2}{4}$$

$$k_{yy}^2 = \frac{a^2}{2} + \frac{t^2}{8} + \frac{l^2}{12}$$

Execution:

a / RUN / t / RUN / k_{xx}^2 / l / RUN / k_{yy}^2

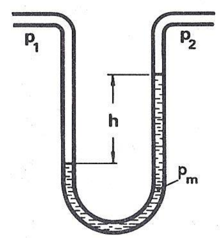
Post-execution (to find volume):

▲▼ / rcl / X / 6·2831852 / = / volume

sto	2	00
X	·	01
+	E	02
(	6	03
stop	0	04
X	·	05
▼	A	06
MEx	5	07
=	—	08
▼	A	09
MEx	5	10
÷	G	11
#	3	12
2	2	13
X	·	14
)	6	15
+	E	16
(	6	17
stop	0	18
X	·	19
▼	A	20
MEx	5	21
=	—	22
▼	A	23
MEx	5	24
X	·	25
÷	G	26
#	3	27
6	6	28
=	—	29
)	6	30
÷	G	31
#	3	32
2	2	33
=	—	34
stop	0	35

PRESSURE FLOW MEASUREMENT

Manometer



pressure difference $p_1 - p_2 = gh(\rho_m - \rho)$

Execution:

ρ_m / RUN / ρ / RUN / h / RUN / pressure difference

In S.I. units; g taken as 9·81 ms^{-2}.

—	F	00
stop	0	01
X	·	02
stop	0	03
X	·	04
#	3	05
9	9	06
·	A	07
8	8	08
1	1	09
=	—	10
stop	0	11
▼	A	12
goto	2	13
0	0	14
0	0	15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

FLOW RATES

Pitot static tube

u = velocity of fluid
P = total pressure
p = static pressure
ρ = density

$$u = \sqrt{\frac{2(P - p)}{\rho}}$$

Execution:

P / RUN / p / RUN / ρ / RUN / u

—	F	00
stop	0	01
÷	G	02
stop	0	03
+	E	04
=	—	05
$\sqrt{x}$	1	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
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		29
		30
		31
		32
		33
		34
		35

FLOW RATES

Sharp edged orifice

A = area
Q = volume flow rate
C_d = discharge coefficient

$$Q = AC_d\sqrt{2gh}$$

Execution:

h / RUN / A / RUN / C_d / RUN / Q

In S.I. units; g taken as $9{\cdot}81ms^{-2}$.

X	·	00
#	3	01
1	1	02
9	9	03
·	A	04
6	6	05
2	2	06
=	—	07
$\sqrt{x}$	1	08
X	·	09
stop	0	10
X	·	11
stop	0	12
=	—	13
stop	0	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

FLOW RATES

Venturi

Subscript 1 refers to tube
Subscript 2 refers to throat
p = static pressure
a = area
u = velocity of fluid
ρ = density

$$u = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[\left(\frac{a_1}{a_2}\right)^2 - 1\right]}}$$

Execution:

a_1 / RUN / a_2 / RUN / ρ / RUN / p_1 / RUN / p_2 / RUN / u

Restrictions:

$a_1 > a_2$, $p_1 > p_2$ or

$a_1 < a_2$, $p_1 < p_2$

÷	G	00
stop	0	01
X	·	02
—	F	03
#	3	04
1	1	05
X	·	06
stop	0	07
÷	G	08
X	·	09
(	6	10
stop	0	11
—	F	12
stop	0	13
+	E	14
)	6	15
=	—	16
$\sqrt{x}$	1	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

PIPE FLOW

L = length
D = diameter
C_f = skin inertia coefficient
ρ = density
U_m = mean velocity

$$\text{pressure drop} = 2\frac{L}{D} C_f \rho U_m^2$$

Execution:

U_m /RUN / ρ / RUN / C_f / RUN / L / RUN / D / RUN / pressure drop

X	·	00
X	·	01
stop	0	02
X	·	03
stop	0	04
X	·	05
stop	0	06
÷	G	07
stop	0	08
+	E	09
=	—	10
stop	0	11
▼	A	12
goto	2	13
0	0	14
0	0	15
		16
		17
		18
		19
		20
		21
		22
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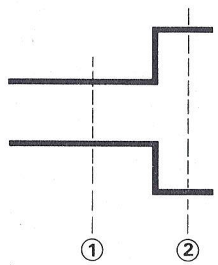
PIPE FLOW

Sudden expansion



Head loss = $\dfrac{(u_1 - u_2)^2}{2g}$ (i)

$$\Delta h = \frac{u_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2 \qquad \text{(ii)}$$

Execution:

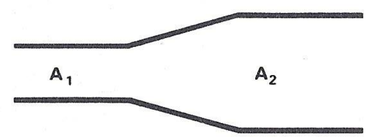
(i) u_1 / RUN / u_2 / RUN / head loss
(post execution with / RUN / RUN / before entering new data)

(ii) u_1 / RUN / RUN / A_1 / RUN / A_2 / RUN / head loss

In S.I. units; g taken as 9·81ms $^{-2}$.

—	F	00
stop	0	01
X	·	02
÷	G	03
#	3	04
1	1	05
9	9	06
·	A	07
6	6	08
2	2	09
X	·	10
(	6	11
stop	0	12
÷	G	13
stop	0	14
—	F	15
#	3	16
1	1	17
X	·	18
)	6	19
=	—	20
stop	0	21
▼	A	22
goto	2	23
0	0	24
0	0	25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

IDEAL PRESSURE RISE DIFFUSER



A = area

$$\Delta p = \frac{\rho u_1^2}{2}\left[1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

Execution:

A_1 / RUN / A_2 / RUN / u_1 / RUN / ρ / RUN / Δp

($A_2 > A_1$ for +ve Δp)

÷	G	00
stop	0	01
X	·	02
—	F	03
#	3	04
1	1	05
—	F	06
X	·	07
(	6	08
stop	0	09
X	·	10
)	6	11
X	·	12
stop	0	13
÷	G	14
#	3	15
2	2	16
=	—	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
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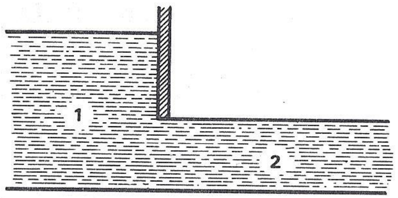
		34
		35

SLUICE GATE



$$F_2^2 = \frac{u_2^2}{gh_2} \qquad \text{(i)}$$

$$= \frac{2h_1^2}{h_2(h_1 + h_2)} \qquad \text{(ii)}$$

Execution:

(i) u_2 / RUN / h_2 / RUN / F_2^2

In S.I. units; g taken as 9·81ms^{-2}.

X	·	00
÷	G	01
stop	0	02
÷	G	03
#	3	04
9	9	05
·	A	06
8	8	07
1	1	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

(ii) h_2 / RUN / h_1 / RUN / F_2^2

÷	G	00
stop	0	01
+	E	02
(	6	03
X	·	04
)	6	05
÷	G	06
+	E	07
=	—	08
▼	A	09
goto	2	10
0	0	11
0	0	12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
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		31
		32
		33
		34
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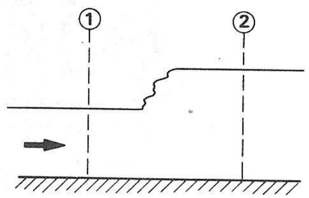
HYDRAULIC JUMP



$$F_1^2 = \frac{h_2(h_1 + h_2)}{2h_1^2} \qquad \text{(i)}$$

$$F_2^2 = \frac{h_1(h_1 + h_2)}{2h_2^2} \qquad \text{(ii)}$$

Execution:

(i) h_2 / RUN / h_1 / RUN / F_1^2

(ii) h_1 / RUN / h_2 / RUN / F_2^2

÷	G	00
stop	0	01
+	E	02
(	6	03
X	·	04
)	6	05
÷	G	06
#	3	07
2	2	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

COMPRESSIBLE FLOW

Perfect gas relationships:

M = mach number.

γ = ratio of specific heats = 1·405 for dry air

$$\frac{T}{T_o} = \left(1 - \frac{(\gamma - 1)\,M^2}{2}\right)$$

$$\frac{P}{P_o} = \left(1 - \frac{(\gamma - 1)\,M^2}{2}\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho}{\rho_o} = \left(1 - \frac{(\gamma - 1)\,M^2}{2}\right)^{\frac{1}{\gamma - 1}}$$

Execution:

M / RUN / γ / RUN / $\frac{T}{T_o}$ / RUN / $\frac{P}{P_o}$ / RUN / $\frac{\rho}{\rho_o}$

X	·	00
—	F	01
(	6	02
X	·	03
stop	0	04
sto	2	05
)	6	06
÷	G	07
#	3	08
2	2	09
+	E	10
#	3	11
1	1	12
=	—	13
stop	0	14
ln	4	15
÷	G	16
(	6	17
rcl	5	18
—	F	19
#	3	20
1	1	21
=	—	22
)	6	23
X	·	24
▼	A	25
MEx	5	26
=	—	27
▼	A	28
e^x	4	29
stop	0	30
rcl	5	31
=	—	32
▼	A	33
e^x	4	34
stop	0	35

THERMODYNAMICS

Polytropic process

p = pressure
v = volume
n = index
T = absolute temperature
R = gas constant

pv^n = constant

To find index or final pressure or volume

(i) $p_2 = p_1 \left(\frac{v_1}{v_2}\right)^n$

(ii) $v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}$

(iii) $n = -\frac{\log\left(\frac{p_2}{p_1}\right)}{\log\left(\frac{v_2}{v_1}\right)}$

Execution:

(i) n / RUN / v_1 / RUN / v_2 / RUN / p_1 / RUN / p_2

(ii) n / ÷ / RUN / p_1 / RUN / p_2 / RUN / v_1 / RUN / v_2

(iii) / ▲▼ / ▲▼ / goto / 1 / 9 / p_1 / RUN / p_2 / RUN / v_1 / RUN / v_2 / RUN / n

X	·	00
(	6	01
stop	0	02
÷	G	03
stop	0	04
=	—	05
ln	4	06
)	6	07
=	—	08
▼	A	09
e^x	4	10
X	·	11
stop	0	12
=	—	13
stop	0	14
▼	A	15
goto	2	16
0	0	17
0	0	18
÷	G	19
stop	0	20
=	—	21
ln	4	22
÷	G	23
(	6	24
stop	0	25
÷	G	26
stop	0	27
=	—	28
ln	4	29
)	6	30
—	F	31
=	—	32
stop	0	33
=	—	34
=	—	35

THERMODYNAMICS

Polytropic process

To find work

$$\text{work} = \frac{p_2 v_2 - p_1 v_1}{1 - n} \qquad \text{(i)}$$

$$= \frac{R(T_2 - T_1)}{1 - n} \qquad \text{(ii)}$$

for a perfect gas

Execution:

(i) p_1 / RUN / v_1 / RUN / p_2 / RUN / v_2 / RUN / n / RUN / work

(ii) R / RUN / T_1 / RUN / RUN / T_2 / RUN / n / RUN / work

sto	2	00
X	·	01
stop	0	02
—	F	03
(	6	04
rcl	5	05
stop	0	06
X	·	07
stop	0	08
)	6	09
÷	G	10
(	6	11
#	3	12
1	1	13
—	F	14
stop	0	15
)	6	16
—	F	17
=	—	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

HEAT CONDUCTION SHAPE FACTORS

Cylinder

r_i = inside radius
r_o = outside radius
L = length
F = shape factor

$$F = \frac{2\pi L}{\ln\left(\frac{r_o}{r_i}\right)}$$

Execution:

r_o / RUN / r_i / RUN / L / RUN / F

÷	G	00
stop	0	01
=	—	02
ln	4	03
÷	G	04
X	·	05
stop	0	06
X	·	07
#	3	08
3	3	09
6	6	10
0	0	11
=	—	12
▼	A	13
D→R	3	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

HEAT CONDUCTION SHAPE FACTORS

Sphere

r_i = inside radius
r_o = outside radius

$$F = \frac{4\pi r_o r_i}{r_o - r_i}$$

Execution:

r_i / RUN / r_o / RUN / F

÷	G	00
—	F	01
(	6	02
stop	0	03
÷	G	04
)	6	05
÷	G	06
X	·	07
#	3	08
7	7	09
2	2	10
0	0	11
=	—	12
▼	A	13
D→R	3	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

HEAT CONDUCTION SHAPE FACTORS

Horizontal disc

r = radius
D = centre line depth

$$F = \frac{2{\cdot}22\,r}{1 - \frac{r}{2{\cdot}83D}}$$

Execution:

r / RUN / D / RUN / F

sto	2	00
÷	G	01
stop	0	02
÷	G	03
#	3	04
2	2	05
·	A	06
8	8	07
3	3	08
—	F	09
#	3	10
1	1	11
—	F	12
÷	G	13
X	·	14
rcl	5	15
X	·	16
#	3	17
2	2	18
·	A	19
2	2	20
2	2	21
=	—	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
0	0	27
		28
		29
		30
		31
		32
		33
		34
		35

HEAT CONDUCTION SHAPE FACTOR

Buried sphere

r = radius
D = centre line depth

$$F = \frac{\pi r}{1 - \frac{r}{2D}}$$

Execution:

r / RUN / D / RUN / F

sto	2	00
÷	G	01
stop	0	02
—	F	03
#	3	04
2	2	05
—	F	06
÷	G	07
X	·	08
rcl	5	09
X	·	10
#	3	11
3	3	12
6	6	13
0	0	14
=	—	15
▼	A	16
D→R	3	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

ACOUSTICS

Adding sound levels (and weighted sound levels) in dB (+ hourly average of sound level) (Log. r.m.s. addition)

$$L_n = 10 \log_{10} \frac{P_n}{P_r} = 4{\cdot}3429448 \ln \frac{P_n}{P_r}$$

where P_r = reference s.p.l. of 2×10^{-5} Nm^{-2}

Definitions:

Noise level addition operator $\oplus$

$$L_m \oplus L_n = 4{\cdot}34294 \ln \left[\exp\left(\frac{L_m}{4{\cdot}34294}\right) + \exp\left(\frac{L_n}{4{\cdot}34294}\right) \right]$$

Noise level subtraction operator $\ominus$

$$L_m \ominus L_n = 4{\cdot}34294 \ln \left[\exp\left(\frac{L_m}{4{\cdot}34294}\right) - \exp\left(\frac{L_n}{4{\cdot}34294}\right) \right]$$

Weighting by time operator $\otimes$

$$L_n \otimes t_n = 4{\cdot}34294 \ln \left[t_n \exp\left(\frac{L_n}{4{\cdot}34294}\right) \right]$$

Averaging over time

$$L_{av} = 4{\cdot}34294 \ln \left[\frac{1}{t} \sum_{k=1}^{n} t_k \exp\left(\frac{L_k}{4{\cdot}34294}\right) \right]$$

$$t = t_1 + t_2 + \cdots + t_n$$

Weighting table ('A' weighting)

f(Hz)	W_f(dB)	f(kHz)	W_f(dB)
31·5	39	1	0
63	26	2	1
125	16	4	1
250	10	8	1
500	3		

÷	G	00
#	3	01
8	8	02
·	A	03
6	6	04
8	8	05
5	5	06
8	8	07
9	9	08
=	—	09
▼	A	10
e^x	4	11
X	·	12
stop	0	13
+	E	14
rcl	5	15
=	—	16
sto	2	17
√x	1	18
ln	4	19
X	·	20
#	3	21
8	8	22
·	A	23
6	6	24
8	8	25
5	5	26
8	8	27
9	9	28
=	—	29
stop	0	30
▼	A	31
goto	2	32
0	0	33
0	0	34
		35

Pre-execution:

/ ▲▼ / ▲▼ / goto / 0 / 0 / C/CE / ▲▼ / sto /
to clear memory

Execution:

(i) Adding noise/sound levels:

L_1 / RUN / RUN / L_2 / RUN / RUN / $L_1 \oplus L_2$ / L_3 / RUN / RUN / $L_1 \oplus L_2 \oplus L_3 \cdots$

(ii) Subtracting noise levels:

L_1 / RUN / RUN / L_2 / RUN / – / RUN / $L_1 \ominus L_2$
(add or subtract levels at will)

(iii) Adding and weighting noise levels:

L_1 / – / W_1 / RUN / RUN / $L_1 - W_1$ / L_2 / – / W_2 / RUN / RUN / $(L_1 - W_1) \oplus (L_2 - W_2) \cdots$ (see table for W_f)

Post execution:

/ ▲▼ / ▲▼ / goto / 1 / 4 / C/CE / ▲▼ / ▲▼ / MEx / ÷ / n / RUN / $L_{weighted}$

where n = no. of levels entered.

(iv) Time averaged noise levels:

L_1 / RUN / X / t_1 / RUN / $L_1 \otimes t_1$ / L_2 / RUN / X / t_2 / RUN / $(L_1 \otimes t_1) \oplus (L_2 \otimes t_2) \cdots$

Post execution:

/ ▲▼ / ▲▼ / goto / 1 / 4 / C/CE / ▲▼ / ▲▼ / MEx / ÷ / t / RUN / L_{av}

(v) Hourly averaged noise level:

Add 24 hourly levels using (i) then post-execution

Post execution:

/ ▲▼ / ▲▼ / goto / 1 / 4 / C/CE / ▲▼ / ▲▼ / MEx / ÷ / 24 / RUN / L_{av}

DECIBEL CONVERSION

$$A_{dB} = 10 \log_{10} \frac{P_2}{P_1} = 20 \log_{10} \frac{E_2}{E_1}$$

$$= 20 \log_{10} \frac{I_2}{I_1}$$

$$P_2 = P_1 \text{ antilog}_{10} \frac{AdB}{10}$$

$$E_2 = E_1 \text{ antilog}_{10} \frac{AdB}{20}$$

$$I_2 = I_1 \text{ antilog}_{10} \frac{AdB}{20}$$

Neper conversion:

$$A_n = \frac{1}{2} \ln \frac{P_2}{P_1} = \ln \frac{E_2}{E_1} = \ln \frac{I_2}{I_1}$$

$$P_2 = P_1 \exp 2A_n$$

$$E_2 = E_1 \exp A_n$$

$$I_2 = I_1 \exp A_n$$

Ratio to dB or nepers:

Execution:

P_2 / ÷ / P_1 / = / ▲▼ / $\sqrt{x}$ /
or E_2 / ÷ / E_1 / = /
or I_2 / ÷ / I_1 / = /
} r / RUN / A_n / RUN / A_{dB}

ln	4	00
stop	0	01
X	·	02
#	3	03
8	8	04
·	A	05
6	6	06
8	8	07
5	5	08
8	8	09
9	9	10
=	—	11
stop	0	12
▼	A	13
goto	2	14
0	0	15
0	0	16
÷	G	17
#	3	18
8	8	19
·	A	20
6	6	21
8	8	22
5	5	23
8	8	24
9	9	25
=	—	26
stop	0	27
▼	A	28
e^x	4	29
X	·	30
stop	0	31
▼	A	32
goto	2	33
1	1	34
7	7	35

dB or nepers to ratio:

Pre-execution:

dB to ratio: ▲▼ / ▲▼ / goto / 1 / 7 / first time only
nepers to ratio: ▲▼ / ▲▼ / goto / 2 / 8 / every time

Execution:

dB to ratio:

A_{dB} / RUN / A_n / RUN / r {
/ X / P_1 / = / P_2
or / E_1 / = / E_2
or / I_1 / = / I_2
or / = / r^2

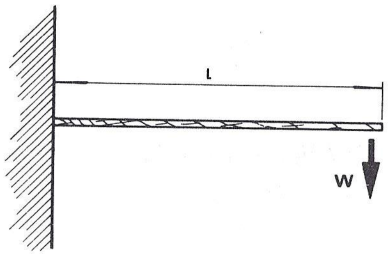
Always use / = / even if no other result is required.

nepers to ratio:

A_n / RUN / r and continue with alternatives as above.

BEAM BENDING

Beam with one fixed end and load W at free end



end slope = $\frac{W\ell^2}{2EI}$

end deflection = $\frac{W\ell^3}{3EI}$

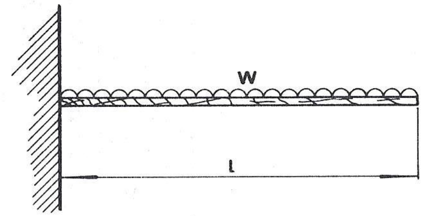
Execution:

ℓ / RUN / W / RUN / E / RUN / I / RUN / slope / RUN / deflection

sto	2	00
X	·	01
X	·	02
stop	0	03
÷	G	04
stop	0	05
÷	G	06
stop	0	07
÷	G	08
#	3	09
2	2	10
÷	G	11
stop	0	12
#	3	13
1	1	14
·	A	15
5	5	16
X	·	17
rcl	5	18
=	—	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

BEAM BENDING

Beam with one fixed end and distributed loading W



end slope = $\frac{W\ell^2}{6EI}$

end deflection = $\frac{W\ell^3}{8EI}$

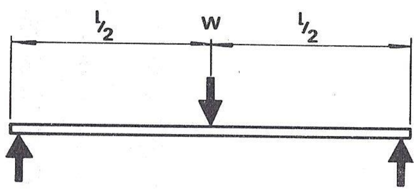
Execution:

ℓ / RUN / W / RUN / E / RUN / I / RUN / slope / RUN / deflection

sto	2	00
X	·	01
X	·	02
stop	0	03
÷	G	04
stop	0	05
÷	G	06
stop	0	07
÷	G	08
#	3	09
6	6	10
X	·	11
stop	0	12
#	3	13
·	A	14
7	7	15
5	5	16
X	·	17
rcl	5	18
=	—	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

BEAM BENDING

Simply supported beam with central load W



end slope = $\frac{W\ell^2}{16EI}$

central deflection = $\frac{W\ell^3}{48EI}$

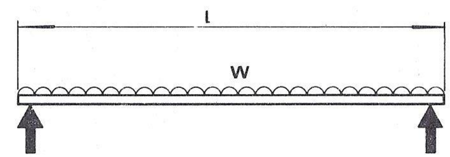
Execution:

ℓ / RUN / W / RUN / E / RUN / I / RUN / end slope / RUN / central deflection

sto	2	00
X	·	01
X	·	02
stop	0	03
÷	G	04
stop	0	05
÷	G	06
stop	0	07
÷	G	08
#	3	09
1	1	10
6	6	11
÷	G	12
stop	0	13
#	3	14
3	3	15
X	·	16
rcl	5	17
=	–	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

BEAM BENDING

Simply supported beam with distributed loading W



end slope = $\frac{W\ell^2}{24EI}$

central deflection = $\frac{5W\ell^3}{384EI}$

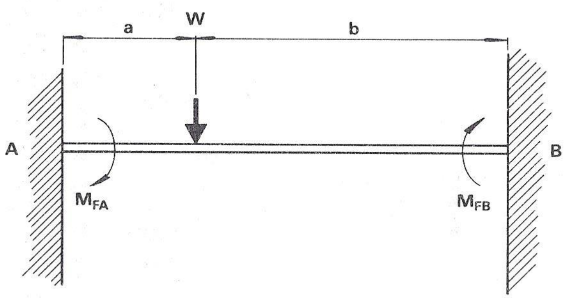
Execution:

ℓ / RUN / W / RUN / E / RUN / I / RUN / end slope / RUN / central deflection

sto	2	00
X	·	01
X	·	02
stop	0	03
÷	G	04
stop	0	05
÷	G	06
stop	0	07
÷	G	08
#	3	09
2	2	10
4	4	11
÷	G	12
stop	0	13
#	3	14
3	3	15
·	A	16
2	2	17
X	·	18
rcl	5	19
=	–	20
stop	0	21
▼	A	22
goto	2	23
0	0	24
0	0	25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

BEAM BENDING

Beam fixed at both ends with load W at a distance from end A



$$M_{FA} = \frac{Wb^2a}{\ell^2}$$

$$M_{FB} = \frac{Wa^2b}{\ell^2}$$

$\ell = a + b$

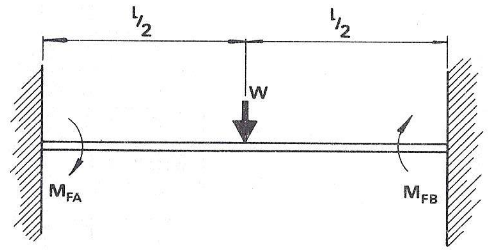
Execution:

b / RUN / a / RUN / W / RUN / ℓ / RUN / M_{FA} / RUN / M_{FB}

sto	2	00
X	·	01
X	·	02
rcl	5	03
X	·	04
(	6	05
stop	0	06
÷	G	07
rcl	5	08
)	6	09
sto	2	10
X	·	11
stop	0	12
÷	G	13
(	6	14
stop	0	15
X	·	16
)	6	17
—	F	18
X	·	19
stop	0	20
rcl	5	21
—	F	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

BEAM BENDING

Beam with two fixed ends and central loading W



Fixed end moments

$$M_{FA} = -\frac{W\ell}{8}$$

$$M_{FB} = \frac{W\ell}{8}$$

Central deflection

$$d = \frac{W\ell^3}{192EI}$$

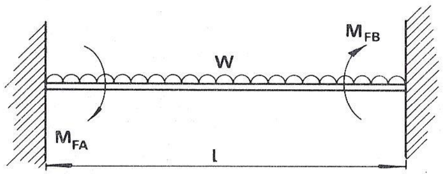
Execution:

W / RUN / ℓ / RUN / E / RUN / I / RUN / M_{FA} / RUN / M_{FB} / RUN / d

÷	G	00
#	3	01
8	8	02
X	·	03
stop	0	04
sto	2	05
X	·	06
(	6	07
▼	A	08
MEx	5	09
X	·	10
)	6	11
÷	G	12
#	3	13
2	2	14
4	4	15
÷	G	16
stop	0	17
÷	G	18
stop	0	19
=	—	20
▼	A	21
MEx	5	22
—	F	23
—	F	24
stop	0	25
=	—	26
stop	0	27
rcl	5	28
stop	0	29
▼	A	30
goto	2	31
0	0	32
0	0	33
		34
		35

BEAM BENDING

Beam between two fixed ends with evenly distributed total load W



Fixed end moments

$$M_{FA} = -\frac{W\ell}{12}$$

$$M_{FB} = \frac{W\ell}{12}$$

Central deflection

$$d = \frac{W\ell^3}{384EI}$$

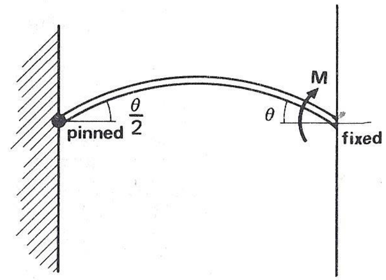
Execution:

W / RUN / ℓ / RUN / E / RUN / I / RUN / M_{FA} / RUN / M_{FB} / RUN / d

÷	G	00
#	3	01
1	1	02
2	2	03
X	·	04
stop	0	05
sto	2	06
X	·	07
(	6	08
▼	A	09
MEx	5	10
X	·	11
)	6	12
÷	G	13
#	3	14
3	3	15
2	2	16
÷	G	17
stop	0	18
÷	G	19
stop	0	20
=	—	21
▼	A	22
MEx	5	23
—	F	24
—	F	25
stop	0	26
=	—	27
stop	0	28
rcl	5	29
stop	0	30
▼	A	31
goto	2	32
0	0	33
0	0	34
		35

BEAM BENDING

Beam with one fixed end, one pinned end. Effect of rotation at fixed end.



M = applied bending moment

$$\text{end slope} = \frac{M\ell}{3EI}$$

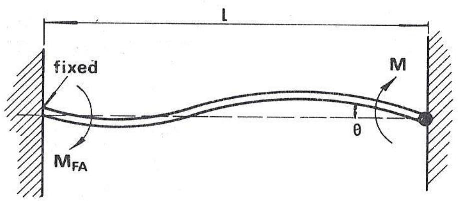
Execution:

M / RUN / ℓ / RUN / E / RUN / I / RUN / end slope

X	·	00
stop	0	01
÷	G	02
stop	0	03
÷	G	04
stop	0	05
÷	G	06
#	3	07
3	3	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

BEAM BENDING

Beam with one fixed end and one pinned end — effect of rotation at pinned end



Moment at fixed end A, = $M_{FA} = \frac{M}{2}$

end slope $\theta = \frac{M\ell}{4EI}$

Execution:

M / RUN / ℓ / RUN / E / RUN / I / RUN / end slope

X	·	00
stop	0	01
÷	G	02
stop	0	03
÷	G	04
stop	0	05
÷	G	06
#	3	07
4	4	08
=	—	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
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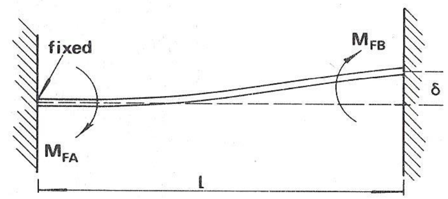
		30
		31
		32
		33
		34
		35

BEAM BENDING

Effect of end displacement on beam fixed at both ends



Moments at fixed ends due to displacement δ

$M_{FA} = M_{FB} = \frac{+6EI\delta}{\ell^2}$

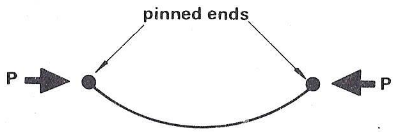
Execution:

E / RUN / I / RUN / δ / RUN / ℓ / RUN / M_{FA}

X	·	00
stop	0	01
X	·	02
stop	0	03
X	·	04
stop	0	05
X	·	06
#	3	07
6	6	08
÷	G	09
(	6	10
stop	0	11
X	·	12
)	6	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

STRUTS

Critical load — strut with two pinned ends



$P_{crit.}$ = critical load = $\frac{\pi^2 EI}{\ell^2}$

Execution:

ℓ / RUN / E / RUN / I / RUN / $P_{crit.}$

÷	G	00
#	3	01
3	3	02
·	A	03
1	1	04
4	4	05
1	1	06
6	6	07
÷	G	08
X	·	09
X	·	10
stop	0	11
X	·	12
stop	0	13
=	—	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

STRUTS

Critical load for strut fixed at one end, pinned at other end

$P_{crit.} = \frac{2\pi^2 EI}{\ell^2}$

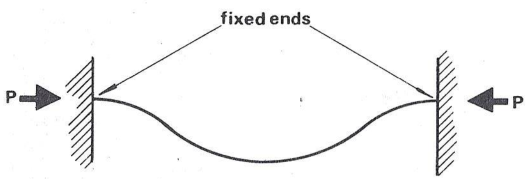
Execution:

ℓ / RUN / E / RUN / I / RUN / $P_{crit.}$

÷	G	00
#	3	01
3	3	02
·	A	03
1	1	04
4	4	05
1	1	06
5	5	07
9	9	08
2	2	09
6	6	10
÷	G	11
X	·	12
+	E	13
X	·	14
stop	0	15
X	·	16
stop	0	17
=	—	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

STRUTS

Strut with two fixed ends



$$P_{crit.} = \frac{4\pi^2 EI}{\ell^2}$$

Execution:

ℓ / RUN / E / RUN / I / RUN / $P_{crit.}$

÷	G	00
#	3	01
6	6	02
·	A	03
2	2	04
8	8	05
3	3	06
2	2	07
÷	G	08
X	·	09
X	·	10
stop	0	11
X	·	12
stop	0	13
=	–	14
stop	0	15
▼	A	16
goto	2	17
0	0	18
0	0	19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

STRUTS

Critical load for strut with one fixed end and one free end

$$P_{crit.} = \frac{EI\pi^2}{(2\ell)^2}$$

Execution:

ℓ / RUN / E / RUN / I / RUN / $P_{crit.}$

÷	G	00
#	3	01
9	9	02
0	0	03
÷	G	04
=	–	05
▼	A	06
D→R	3	07
X	·	08
X	·	09
stop	0	10
X	·	11
stop	0	12
=	–	13
stop	0	14
▼	A	15
goto	2	16
0	0	17
0	0	18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

TORSION OF THIN WALLED TUBE

$$\text{Torque} = 2\pi r^3 t\, G \frac{\theta}{L}$$

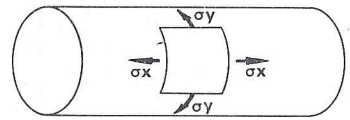
$$\frac{\theta}{L} = \text{twist per unit length} = \frac{\text{angular deflection}}{\text{length}}$$

Execution:

r / RUN / t / RUN / G / RUN / $\frac{\theta}{L}$ / RUN / torque

X	·	00
(	6	01
X	·	02
)	6	03
X	·	04
#	3	05
3	3	06
6	6	07
0	0	08
X	·	09
stop	0	10
X	·	11
stop	0	12
X	·	13
stop	0	14
=	–	15
▼	A	16
D→R	3	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

CYLINDRICAL PRESSURE VESSEL



Longitudinal stress $\sigma_x = \frac{pd}{4t}$

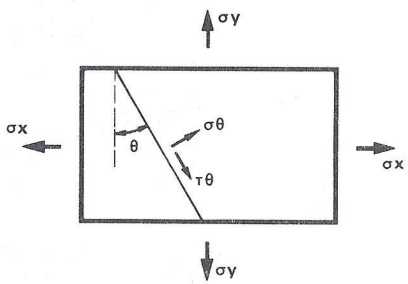
Hoop stress $\sigma_y = \frac{pd}{2t}$

Execution:

p / RUN / d / RUN / t / RUN / σ_x / RUN / σ_y

X	·	00
stop	0	01
÷	G	02
stop	0	03
÷	G	04
#	3	05
4	4	06
+	E	07
stop	0	08
=	–	09
stop	0	10
▼	A	11
goto	2	12
0	0	13
0	0	14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

COMPLEX STRESSES



$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

Execution:

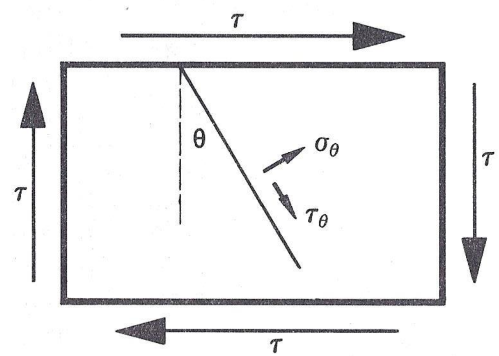
σ_x / + / σ_y / RUN / θ / RUN / σ_θ / θ / ▲▼ / ▲▼ / MEx/ RUN / τ_θ

For angle θ in degrees use / ▲▼ / ▲▼ / D→R / after entering θ each time.

For negative θ use / – / = / after third / RUN / to give correct sign of τ_θ.

sto	2	00
÷	G	01
#	3	02
2	2	03
+	E	04
(	6	05
–	F	06
rcl	5	07
=	–	08
sto	2	09
stop	0	10
sin	7	11
X	·	12
+	E	13
–	F	14
+	E	15
#	3	16
1	1	17
X	·	18
rcl	5	19
)	6	20
=	–	21
stop	0	22
X	·	23
(	6	24
rcl	5	25
sin	7	26
)	6	27
X	·	28
(	6	29
rcl	5	30
cos	8	31
)	6	32
+	E	33
=	–	34
stop	0	35

COMPLEX STRESSES



$\sigma_\theta = \tau \sin 2\theta$

$\tau_\theta = -\tau \cos 2\theta$

Execution:

θ / RUN / τ / RUN / τ_θ / RUN / σ_θ

For θ in degrees insert / ▼ / D→R / at start of program or use / ▲▼ / ▲▼ / D→R / after entering θ.

For negative θ, use / – / = / after third / RUN / to give correct sign of σ_θ.

sin	7	00
X	·	01
+	E	02
–	F	03
+	E	04
#	3	05
1	1	06
=	–	07
sto	2	08
stop	0	09
X	·	10
(	6	11
X	·	12
rcl	5	13
–	F	14
=	–	15
stop	0	16
rcl	5	17
X	·	18
–	F	19
+	E	20
#	3	21
1	1	22
=	–	23
$\sqrt{x}$	1	24
)	6	25
=	–	26
stop	0	27
▼	A	28
goto	2	29
0	0	30
0	0	31
		32
		33
		34
		35

ELASTIC STRAIN ENERGY

Elastic strain energy:

(i) In tension $\dfrac{\sigma^2}{2E}$

(ii) In torsion $\dfrac{\tau^2}{2G}$

Execution:

(i) σ / RUN / E / RUN / energy

(ii) τ / RUN / G / RUN / energy

X	·	00
÷	G	01
#	3	02
2	2	03
÷	G	04
stop	0	05
=	—	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
		12
		13
		14
		15
		16
		17
		18
		19
		20
		21
		22
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		31
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		33
		34
		35

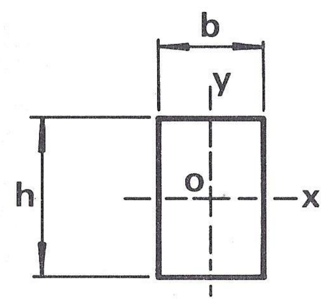
ELASTIC AND PLASTIC SECTIONAL MODULI

Z_e = elastic section modulus

Z_p = plastic section modulus

shape factor $S = \dfrac{Z_p}{Z_e}$

Solid rectangular section



Axis C_y: $Z_e = \dfrac{b^2h}{6}$

$Z_p = \dfrac{b^2h}{4}$

S = 1·5

Axis C_z: $Z_e = \dfrac{bh^2}{6}$

$Z_p = \dfrac{bh^2}{4}$

S = 1·5

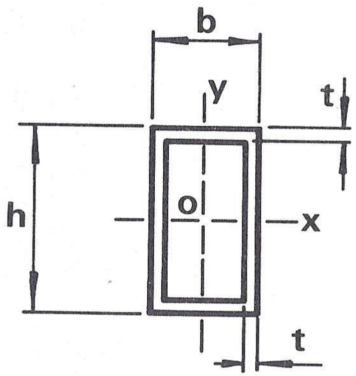
Execution:

b / RUN / h / RUN / Z_e for C_y / RUN / Z_p for C_y / RUN / Z_e for C_z / RUN / Z_p for C_z

sto	2	00
X	·	01
X	·	02
stop	0	03
÷	G	04
#	3	05
6	6	06
X	·	07
stop	0	08
#	3	09
1	1	10
·	A	11
5	5	12
÷	G	13
stop	0	14
rcl	5	15
X	·	16
÷	G	17
rcl	5	18
÷	G	19
#	3	20
·	A	21
3	3	22
7	7	23
5	5	24
X	·	25
stop	0	26
#	3	27
1	1	28
·	A	29
5	5	30
=	—	31
stop	0	32
=	—	33
=	—	34
=	—	35

ELASTIC AND PLASTIC SECTIONAL MODULI

Thin walled rectangular box



(t small compared to h and b)

Axis C_y: $Z_e = bt\left(h + \frac{b}{3}\right)$

$$Z_p = bt\left(h + \frac{b}{2}\right)$$

$$S = \frac{h + \frac{b}{2}}{h + \frac{b}{3}}$$

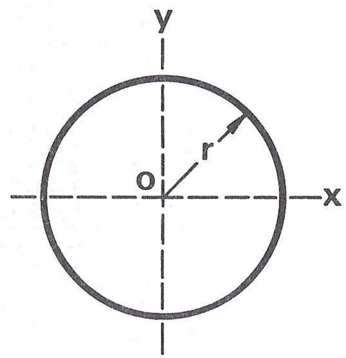
Execution:

h / RUN / b / RUN / t / RUN / Z_e / RUN / Z_p / RUN / S

÷	G	00
stop	0	01
sto	2	02
+	E	03
#	3	04
·	A	05
5	5	06
=	—	07
▼	A	08
MEx	5	09
X	·	10
X	·	11
stop	0	12
X	·	13
(	6	14
#	3	15
6	6	16
÷	G	17
—	F	18
+	E	19
rcl	5	20
÷	G	21
▼	A	22
MEx	5	23
÷	G	24
=	—	25
▼	A	26
MEx	5	27
)	6	28
X	·	29
stop	0	30
rcl	5	31
=	—	32
stop	0	33
rcl	5	34
stop	0	35

ELASTIC AND PLASTIC SECTIONAL MODULI

Solid circular section



$$Z_e = \frac{\pi r^3}{4}$$

$$Z_p = \frac{4r^3}{3}$$

$$S = \frac{16}{3\pi} = 1{\cdot}697653$$

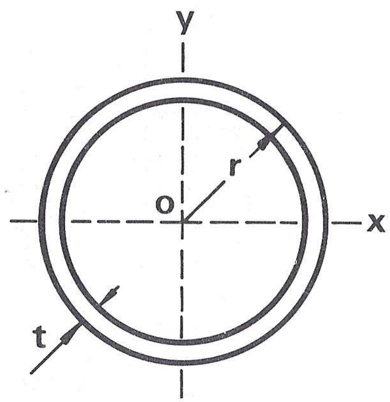
Execution:

r / RUN / Z_e / RUN / Z_p

X	·	00
(	6	01
X	·	02
)	6	03
X	·	04
sto	2	05
#	3	06
4	4	07
5	5	08
=	—	09
▼	A	10
D→R	3	11
stop	0	12
rcl	5	13
÷	G	14
#	3	15
·	A	16
7	7	17
5	5	18
=	—	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

ELASTIC AND PLASTIC SECTIONAL MODULI

Thin walled circular tube



(t small compared to r)

$Z_e = \pi r^2 t$

$Z_p = 4r^2 t$

$S = \dfrac{4}{\pi} = 1{\cdot}273240$

Execution:

r / RUN / t / RUN / Z_e / RUN / Z_p

X	·	00
X	·	01
stop	0	02
X	·	03
sto	2	04
#	3	05
1	1	06
8	8	07
0	0	08
=	—	09
▼	A	10
D→R	3	11
stop	0	12
rcl	5	13
+	E	14
+	E	15
=	—	16
stop	0	17
▼	A	18
goto	2	19
0	0	20
0	0	21
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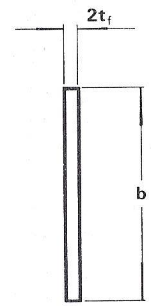
ELASTIC AND PLASTIC SECTIONAL MODULI

Thin I-section



(thickness small compared to overall dimensions)

Axis C_y : $Z_e = \dfrac{b^2 t_f}{3}$

$Z_p = \dfrac{b^2 t_f}{2}$

$S = 1{\cdot}5$

Execution:

b / RUN / t_f / RUN / Z_e / RUN / Z_p

X	·	00
X	·	01
stop	0	02
÷	G	03
#	3	04
3	3	05
X	·	06
stop	0	07
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·	A	10
5	5	11
=	—	12
stop	0	13
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goto	2	15
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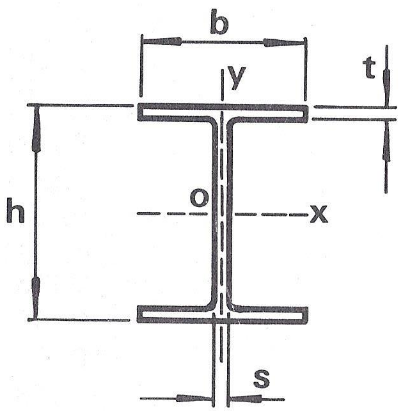
ELASTIC AND PLASTIC SECTIONAL MODULI

Thin I-section



Axis C_z: $$Z_e = h\left(bt + \frac{hs}{6}\right)$$

$$Z_p = h\left(bt + \frac{hs}{4}\right)$$

$$S = \frac{bt + \frac{hs}{4}}{bt + \frac{hs}{6}}$$

Execution:

h / RUN / S / RUN / b / RUN / t / RUN / Z_e / RUN / Z_p / RUN / S

sto	2	00
X	·	01
stop	0	02
÷	G	03
#	3	04
1	1	05
2	2	06
=	—	07
▼	A	08
MEx	5	09
X	·	10
(	6	11
stop	0	12
X	·	13
stop	0	14
+	E	15
rcl	5	16
+	E	17
rcl	5	18
+	E	19
▼	A	20
MEx	5	21
÷	G	22
rcl	5	23
=	—	24
▼	A	25
MEx	5	26
)	6	27
X	·	28
stop	0	29
rcl	5	30
=	—	31
stop	0	32
rcl	5	33
stop	0	34
=	—	35

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4

CONTENTS

How to use these programs

Each program is arranged as follows:

1. On the left of the page, explanatory information and the 'execution sequence', the sequence of keystrokes necessary for running the program. Results displayed are printed in gold.
2. In the first column on the right hand side of the page, the sequence of keystrokes which make up the program.
3. In the second and third columns on the right hand side of the page, the program in check symbol and step number form (see section on checking the program).

Notes

1. Where a key has more than one function, the relevant function is printed as the keystroke in the first column

   e.g. the keystroke [8] (cos / arccos) may appear as 8, cos or arccos.
2. The symbol ▾ within a program always refers to the key [·/EE/–] (▼)
3. The symbol # refers to [3] (ChN/#)
4. The abbreviation gin is 'go if neg' and so refers to the key [1] (go if neg)

Entering the program

To enter a program into the calculator:

1. Press [▲▼] [▲▼] [2] (go to) [0] [0]	Display shows step programmed at 00 in check symbol form as described below.
2. Press [▲▼] [RUN] (learn)	No change in display.
3. Press the sequence of keys for the program as shown in the first column of the program page.	At each stage the step about to be overwritten is displayed. When the machine is first switched on every step is zero.
4. Press [C/CE]	Normal number display is resumed.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]	The step programmed at 00 will be displayed.

Checking the program

Each of the programs in the library is shown in check symbol form in the second column on the right-hand side of the page.

Press [▲▼] [C/CE] (step) repeatedly, and at each stage the check symbol will appear on the left of the display with the step number on the right. Ignore the four zeros in the display.

e.g. A.0000 03 (check symbol: A; step number: 03)

After stepping through the program, press

[▲▼] [▲▼] [2] (go to) [0] [0] before execution.

Finally, press [C/CE] and the program is ready for use.

Correcting the program

If the check symbol for a particular step number is not as indicated in the last two columns of the program page:

1. Press [▲▼] [▲▼] [2] (go to)

   followed by the step number if the appropriate step number is not already displayed.
2. Press [▲▼] [RUN] (learn)
3. Enter the correct keystroke. The display will then show the next step in the program. If this is also incorrect, enter the correct keystroke. At each stage, the step about to be overwritten will be displayed.
4. When correction has been completed, press [C/CE]. Any step which has not been overwritten will not be affected.
5. Press [▲▼] [▲▼] [2] (go to) [0] [0]

Note

To restore normal use of the calculator after entering or checking the program, press [C/CE]

Running the program

Press the sequence of keys as shown in the program library in the execution sequence. Results displayed are printed in gold.

REACTANCES AND IMPEDANCES

Introduction

General note: conventions:

Voltage transfer ratios and current transfer ratios denoted by a_v and a_i are positive fractions $0 \leqslant a \leqslant 1$

Expressed in dB as gain, $A = 20 \log a$ is –ve

When expressed as an attenuation in dB, A is +ve and is given by $A = -20 \log a$

Power gain $= a_v a_i = a^2$, so $A = 10 \log (a^2) = 20 \log a$

Characteristic or design impedance $= R_o$

RESISTORS IN PARALLEL

(capacitors in series)
(inductors in parallel)
(conductors in series)

÷	G	00
+	E	01
rcl	5	02
=	–	03
sto	2	04
÷	G	05
=	–	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
		12
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Pre-execution:

0 / ▲▼ / sto / C/CE / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

R_1 / RUN / R_2 / RUN / $\frac{R_1 R_2}{R_1 + R_2}$ / R_3 / · · · / R_n / RUN / $R_{parallel}$

Alternative execution:

To find resistor R_2 required to make parallel combination of R_1 and $R_2 = R$:

R / RUN / R_1 / ▲▼ / ▲▼ / +/– / RUN / R_2

(R_1 must be greater than R)

REACTANCE – FREQUENCY CONVERSIONS

$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$ (i)

$X_L = 2\pi fL = \omega L$ (ii)

$C = \frac{1}{2\pi fX_C} = \frac{1}{\omega X_C}$ (iii)

$L = \frac{X_L}{2\pi f} = \frac{X_L}{\omega}$ (iv)

$f = \frac{1}{2\pi CX_C}$ (v)

$f = \frac{X_L}{2\pi L}$ (vi)

Execution:

f / RUN / { ÷ / RUN / ω
or C / ÷ / RUN / X_C (i)
or L / RUN / X_L (ii)
or X_C / ÷ / RUN / C (iii)
or ÷ / X_L / RUN / L (iv) }

C / RUN / X_C / ÷ / RUN / f (v)

L / RUN / ÷ / X_L / RUN / f (vi)

X	·	00
#	3	01
6	6	02
·	A	03
2	2	04
8	8	05
3	3	06
1	1	07
8	8	08
5	5	09
3	3	10
÷	G	11
÷	G	12
stop	0	13
÷	G	14
=	–	15
stop	0	16
▼	A	17
goto	2	18
0	0	19
0	0	20
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MAGNITUDE AND PHASE OF IMPEDANCE

$Z = R + jX = |Z|e^{j\phi}$

$|Z| = \sqrt{R^2 + X^2}$ $\phi = \arctan\left(\frac{X}{R}\right)$

Execution:

X / RUN / R / RUN / |Z| / RUN / ϕ

For ϕ in degrees, insert / ▼ / R→D / after step 19.

sto	2	00
X	·	01
+	E	02
(	6	03
stop	0	04
÷	G	05
▼	A	06
MEx	5	07
÷	G	08
=	–	09
▼	A	10
arctan	9	11
▼	A	12
MEx	5	13
X	·	14
)	6	15
=	–	16
√x	1	17
stop	0	18
rcl	5	19
stop	0	20
▼	A	21
goto	2	22
0	0	23
0	0	24
		25
		26
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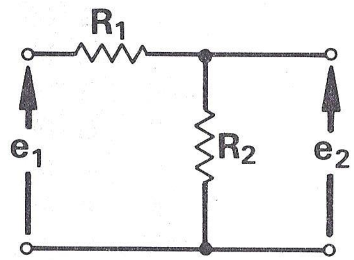
RESISTIVE VOLTAGE DIVIDER



To find R_1, R_2 given $R = R_1 + R_2$ and a or A

where $a = \frac{e_2}{e_1}$ $A = 20 \log \frac{e_2}{e_1}$

Execution:

R / RUN / a / RUN / R_2 / RUN / R_1

If A rather than a is given, see program on page 13.

—	F	00
(	6	01
X	·	02
stop	0	03
)	6	04
stop	0	05
=	—	06
stop	0	07
▼	A	08
goto	2	09
0	0	10
0	0	11
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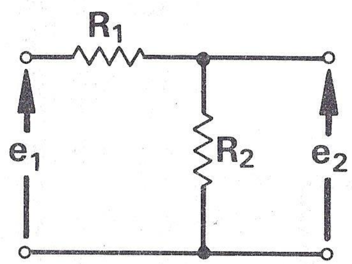
RESISTIVE VOLTAGE DIVIDER



Given total resistance and attenuation, to find resistor values:

$R = R_1 + R_2$

$a = \frac{e_2}{e_1}$, $A = 20 \log \frac{e_2}{e_1}$ dB

Execution:

R / RUN / A / RUN / a / RUN / R_2 / RUN / R_1 / RUN / A / RUN / a / RUN / R_2 / RUN / R_1 / RUN / A / · · ·

If a is given, execute as below, or see shorter program on page 12.

R / RUN / ▲▼ / ▲▼ / goto / 13 / a / RUN / R_2 / RUN / R_1 / RUN / ▲▼ / ▲▼ / goto / 13 / a / RUN / R_2 / · · ·

sto	2	00
stop	0	01
÷	G	02
#	3	03
8	8	04
·	A	05
6	6	06
8	8	07
5	5	08
8	8	09
9	9	10
—	F	11
=	—	12
▼	A	13
e^x	4	14
stop	0	15
X	·	16
rcl	5	17
—	F	18
stop	0	19
rcl	5	20
—	F	21
=	—	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
2	2	27
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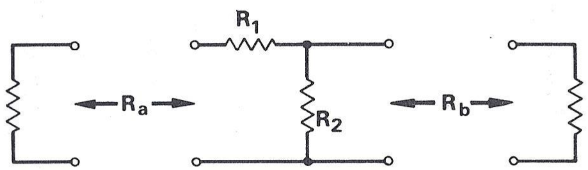
RESISTIVE L—PAD MATCHING IMPEDANCES



$$R_1 = \sqrt{R_a(R_a - R_b)} \qquad R_2 = \frac{R_a R_b}{R_1}$$

$$a_v = \frac{R_a - R_1}{R_a} \qquad A_v = 20 \log a_v$$

$$a_i = \frac{R_a}{R_a + R_1} \qquad A_i = 20 \log a_i$$

$$g = a_v a_i \qquad G = 10 \log a_v a_i$$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / if previous run incomplete

sto	2	00
X	·	01
—	F	02
(	6	03
stop	0	04
X	·	05
rcl	5	06
)	6	07
sto	2	08
=	—	09
√X	1	10
stop	0	11
÷	G	12
X	·	13
rcl	5	14
X	·	15
stop	0	16
÷	G	17
X	·	18
rcl	5	19
+	E	20
sto	2	21
#	3	22
1	1	23
=	—	24
√X	1	25
—	F	26
(	6	27
▼	A	28
MEx	5	29
√X	1	30
)	6	31
÷	G	32
stop	0	33
=	—	34
stop	0	35

Execution:

R_a / RUN / R_b / RUN / R_1 / RUN / R_2 / RUN / $\sqrt{g}$ / and continue as required with one of the following sequences:

(i) To find a_v, A_v, A_i, G:

▲▼ / ▲▼ / MEx / RUN / a_v / ▲▼ / ln / X / 8·68589 / = / A_v

▲▼ / ▲▼ / MEx / ▲▼ / ln / X / 8·68589 / + / G

/ — / ▲▼ / rcl / = / A_i *or*

(ii) To find a_v:

/ ▲▼ / rcl / RUN / a_v *or*

(iii) To find a_i:

/ 1 / X / ▲▼ / rcl / RUN / a_i *or*

(iv) To find g:

/ 1 / X / RUN / g *or*

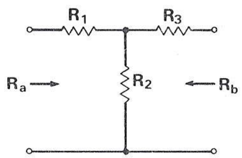
(v) To find a_v, g, a_1:

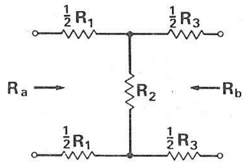
/ ▲▼ / ▲▼ / MEx / RUN / a_v

/ ▲▼ / ▲▼ / MEx / X / = / g

/ ÷ / ▲▼ / rcl / = / a_i

RESISTIVE ATTENUATOR SECTIONS, T–TYPE





Unbalanced T-network   Balanced H-network

$R_o = \sqrt{R_a R_b}\,, \qquad \rho = \dfrac{R_a}{R_o} = \dfrac{R_o}{R_b}$

Design attenuation = a (<1) = $\sqrt{a_v a_i}$

Power attenuation = A = –20 log a

Forward voltage transfer ratio $a_v = \dfrac{a}{\rho}$

Forward current transfer ratio $a_i = a\rho$

$$R_1 = \left[\frac{\rho(1 + a^2) - 2a}{1 - a^2}\right] R_o = (\rho k_1 - k_2)\,R_o$$

$$R_3 = \left[\frac{\frac{1}{\rho}(1 + a^2) - 2a}{1 - a^2}\right] R_o = \left(\frac{1}{\rho}k_1 - k_2\right) R_o$$

$$R_2 = \left[\frac{2a}{1 - a^2}\right] R_o = k_2 R_o$$

X	·	00
(	6	01
X	·	02
–	F	03
+	E	04
#	3	05
1	1	06
=	–	07
sto	2	08
÷	G	09
)	6	10
+	E	11
X	·	12
stop	0	13
–	F	14
stop	0	15
+	E	16
(	6	17
#	3	18
2	2	19
–	F	20
rcl	5	21
÷	G	22
rcl	5	23
X	·	24
stop	0	25
)	6	26
sto	2	27
=	–	28
stop	0	29
X	·	30
÷	G	31
X	·	32
rcl	5	33
–	F	34
stop	0	35

Pre-execution: use as required:

(i) given R_a and R_b, find and note ρ and R_o

R_a / ▲▼ / sto / ÷ / R_b / = / ▲▼ / $\sqrt{x}$ / ρ / ÷ / X / ▲▼ / rcl / = / R_o

(ii) given A, find and note a

A / – / ÷ / 8·68589 / = / ▲▼ / ▲▼ / e^x / a

▲▼ / ▲▼ / goto / 0 / 0 /

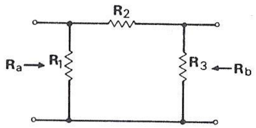
Execution:

a / RUN / k_2 / R_o / RUN / R_2 / RUN / k_1 / R_o / X / ρ / RUN / R_1 / ρ / RUN / R_2 / = / R_3

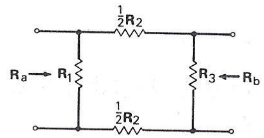
Special case, ρ = 1:

a / RUN / k_2 / R_o / RUN / R_2 / RUN / k_1 / R_o / RUN / R_1 = R_3

RESISTIVE ATTENUATOR SECTIONS, π TYPE



Unbalanced π section



Balanced O section

$R_o = \sqrt{R_a R_b}$ $\qquad \rho = \dfrac{R_a}{R_o} = \dfrac{R_o}{R_b}$

a_v = forward voltage transfer ratio = $\dfrac{a}{\rho}$

a_i = forward current transfer ratio = $a\rho$

a = design attenuation = $\sqrt{a_v a_i}$

A = power attenuation = –20 log a (in dB)

$$R_1 = \left[\frac{1 - a^2}{\frac{1}{\rho}(1 + a^2) - 2a}\right] R_o$$

$$R_3 = \left[\frac{1 - a^2}{\rho\,(1 + a^2) - 2a}\right] R_o$$

$$R_2 = \left[\frac{1 - a^2}{2a}\right] R_o$$

Pre-execution (as required):

(i) calculate and note ρ:
R_a / ▲▼ / sto / ÷ / R_b / = / ▲▼ / $\sqrt{x}$ / ρ
and continue to find R_o:
/ ÷ / X / ▲▼ / rcl / = / R_o

(ii) find and note a if given A:
/ A / – / ÷ / 8·68589 / = / ▲▼ / ▲▼ / e^x / a

set program:
▲▼ / ▲▼ / goto / 0 / 0 /

X	·	00
(	6	01
X	·	02
–	F	03
+	E	04
#	3	05
1	1	06
=	–	07
sto	2	08
÷	G	09
)	6	10
+	E	11
÷	G	12
X	·	13
stop	0	14
÷	G	15
stop	0	16
–	F	17
+	E	18
(	6	19
#	3	20
2	2	21
–	F	22
rcl	5	23
÷	G	24
rcl	5	25
÷	G	26
stop	0	27
)	6	28
sto	2	29
÷	G	30
=	–	31
stop	0	32
=	–	33
=	–	34
=	–	35

Execution:

a / RUN / R_o / RUN / R_2 / RUN / ρ / ÷ / R_o / RUN / R_1

Post-execution:

ρ / X / X / ▲▼ / rcl / – / ▲▼ / (/ R_2 / ÷ / ▲▼ /) / ÷ / = / R_3

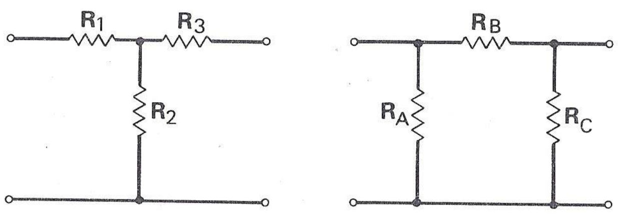
Special case : ρ = 1:

Execution:

a / RUN / R_o / RUN / R_2 / RUN / R_o / RUN / $R_1 = R_3$

RESISTOR NETWORKS

Π to T and T to Π transformations



$$R_o^2 = \frac{R_A R_B R_C}{R_A + R_B + R_C} = R_1 R_2 + R_2 R_3 + R_3 R_1$$

$$R_1 R_C = R_2 R_B = R_3 R_A = R_o^2$$

Execution:

(i) R_o known:

R_o / X / = / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

(ii) Π to T:

▲▼ / ▲▼ / goto / 0 / 9 / R_A / RUN / R_B / RUN / R_C / RUN / RUN /

(ii) T to Π:

▲▼ / ▲▼ / goto / 0 / 9 / R_1 / ÷ / RUN / R_2 / ÷ / RUN / R_3 / ÷ / RUN / ÷ / RUN /

Follow any of (i), (ii) or (iii) with either:

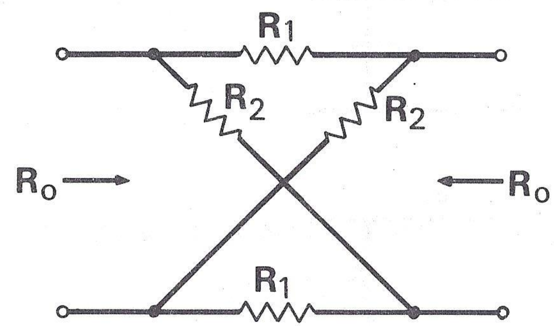
R_A / RUN / R_3 / R_B / RUN / R_2 / R_C / RUN / R_1

or:

R_1 / RUN / R_C / R_2 / RUN / R_B / R_3 / RUN / R_A

÷	G	00
X	·	01
rcl	5	02
=	—	03
stop	0	04
▼	A	05
goto	2	06
0	0	07
0	0	08
X	·	09
sto	2	10
(	6	11
stop	0	12
+	E	13
rcl	5	14
—	F	15
▼	A	16
MEx	5	17
)	6	18
X	·	19
(	6	20
stop	0	21
+	E	22
rcl	5	23
—	F	24
▼	A	25
MEx	5	26
)	6	27
÷	G	28
rcl	5	29
stop	0	30
=	—	31
sto	2	32
stop	0	33
=	—	34
=	—	35

LATTICE ATTENUATOR SECTIONS



(must be balanced, constant impedance)

$a_v = a_i = a$ $\qquad$ $A = -20 \log a$

Characteristic impedance = R_o

$$R_1 = \frac{1-a}{1+a} R_o \qquad R_2 = \frac{1+a}{1-a} R_o$$

Execution:

either

/ ▲▼ / ▲▼ / goto / 1 / 3 / a / RUN / R_o / RUN / R_2 / RUN / R_1

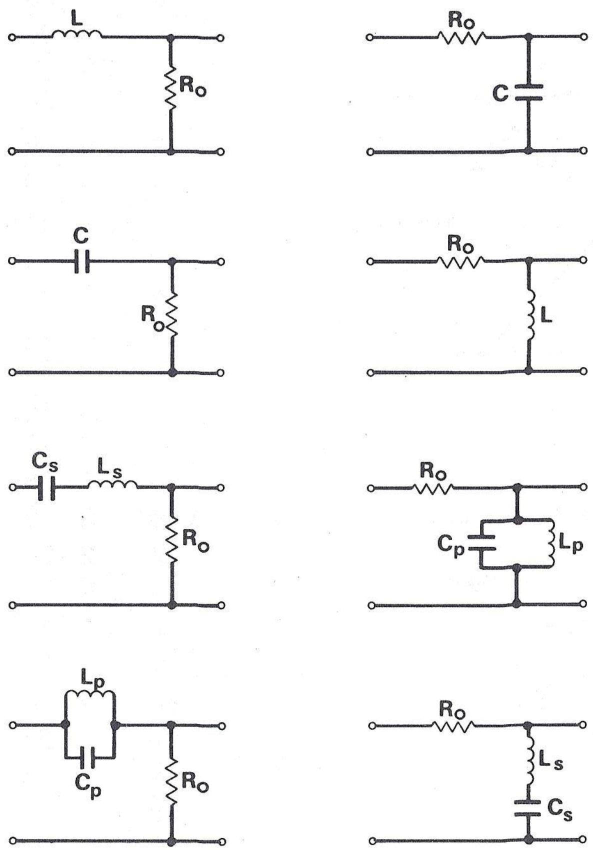
or

/ A / RUN / R_o / RUN / R_2 / RUN / R_1

—	F	00
÷	G	01
#	3	02
8	8	03
·	A	04
6	6	05
8	8	06
5	5	07
8	8	08
9	9	09
=	—	10
▼	A	11
e^x	4	12
+	E	13
#	3	14
1	1	15
÷	G	16
(	6	17
—	F	18
#	3	19
2	2	20
—	F	21
)	6	22
X	·	23
sto	2	24
stop	0	25
=	—	26
stop	0	27
÷	G	28
(	6	29
rcl	5	30
X	·	31
)	6	32
=	—	33
stop	0	34
=	—	35

FILTERS

Simple filters



Normalised to design impedance R_o,
ω_o = cut-off angular frequency (low-pass or high pass)

ω_o = centre frequency (band-pass or band stop)
ω_2 = upper cut-off frequency (band-pass or band stop)
ω_1 = lower cut-off frequency (band-pass or band stop)

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

$$n = \frac{\omega_2 - \omega_1}{\omega_o}$$

Definitions:

$$x = \text{normalised frequency parameter} = \frac{\omega}{\omega_o}$$

$$v = \text{deviation parameter} = x \text{ (low pass)} = -\frac{1}{x} \text{ (high pass)}$$

$$v = \frac{x - \frac{1}{x}}{n} \text{ (band pass)} \quad = \frac{n}{\frac{1}{x} - x} \text{ (band stop)}$$

Design:

Low-pass and high pass:

$$L = \frac{R_o}{\omega_o} \qquad C = \frac{1}{\omega_o R_o}$$

Use frequency-reactance conversion program (page 10)

Band-pass and band stop:

$$\omega_o \sqrt{L_p C_p} = \omega_o \sqrt{L_s C_s} = 1$$

$$L_s = \frac{L}{n}, \quad C_s = nC \qquad L_p = nL, \quad C_p = \frac{C}{n}$$

Use frequency-reactance conversion program (page 10)

FILTERS

Simple filters (contd.)

Performance:

A = attenuation (dB) = $-8{\cdot}68589 \ln \sqrt{1+v^2}$

ϕ = phase = $-\arctan v$

Execution:

Band-pass:

x / RUN / n / RUN / v / RUN / A / RUN / ϕ

Band stop:

x / RUN / n / ÷ / — / RUN / v / RUN / A / RUN / ϕ

Low pass:

▲▼ / ▲▼ / goto / 1 / 0 / x / RUN / A / RUN / ϕ (v = x)

High pass:

▲▼ / ▲▼ / goto / 0 / 8 / x / ÷ / — / RUN / v / RUN / A / RUN / ϕ

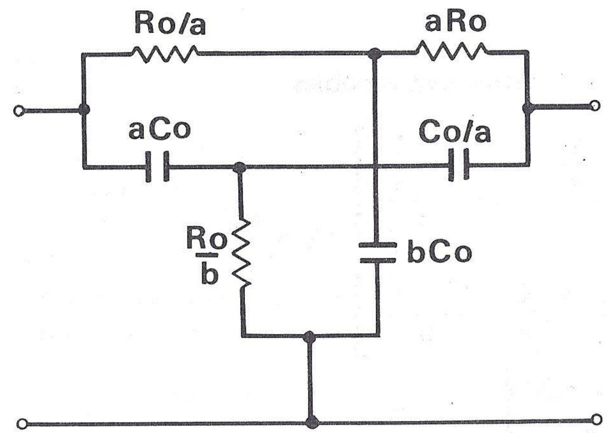
To obtain x, pre-execution could be:

f / ÷ / f_o / = / or ω / ÷ / ω_o / = /

sto	2	00
—	F	01
(	6	02
rcl	5	03
÷	G	04
)	6	05
÷	G	06
stop	0	07
=	—	08
stop	0	09
sto	2	10
X	·	11
+	E	12
#	3	13
1	1	14
=	—	15
$\sqrt{x}$	1	16
ln	4	17
—	F	18
X	·	19
#	3	20
8	8	21
·	A	22
6	6	23
8	8	24
5	5	25
8	8	26
9	9	27
=	—	28
stop	0	29
rcl	5	30
▼	A	31
arctan	9	32
—	F	33
=	—	34
stop	0	35

FILTERS

The twin-T network



Design:

ω_o = null frequency $\qquad x = \dfrac{\omega}{\omega_o}$

$\omega_o C_o R_o = 1$ (use reactance frequency program)

$b = a + \dfrac{1}{a} \qquad v = -\dfrac{n}{x - \dfrac{1}{x}}$

$u = \dfrac{x - \dfrac{1}{x}}{b}$, where $n = \dfrac{2b}{a} = 2 + \dfrac{2}{a^2}$

$G_o = \dfrac{1}{R_o} \qquad a = \sqrt{\dfrac{2}{n-2}}$

FILTERS

The twin-T network (contd.)

Performance:

The Y-matrix is, in terms or normalised variables:

$$Y = \frac{G_o}{1 + jx} \begin{bmatrix} 2a + j\dfrac{x - \frac{1}{x}}{b} & -j\dfrac{x - \frac{1}{x}}{b} \\ -j\dfrac{x - \frac{1}{x}}{b} & \dfrac{2}{a} + j\dfrac{x - \frac{1}{x}}{b} \end{bmatrix} \quad \Delta Y = \frac{2G_o^2}{jx}$$

$$= \frac{G_o}{1 + jx} \begin{bmatrix} 2a + ju & -ju \\ -ju & \dfrac{2}{a} + ju \end{bmatrix}$$

with zero source impedance and load admittance (the usual conditions)

$$a_v = -\frac{y_{21}}{y_{22}} = \frac{ju}{\frac{2}{a} + ju} = \frac{1}{1 + jv} = -\frac{2b}{a\left(x - \frac{1}{x}\right)}$$

Attenuation in dB = A = $-8{\cdot}68589 \ln \sqrt{1 + v^2}$

Phase, $\phi = -\arctan v$

Use simple filters program with $n = \frac{2b}{a}$ (see page 24)

FILTERS

The twin-T network (contd.)

Design case:

Given:

Lower cut-off frequency = ω_1,
null frequency = ω_0.

Find:

a, hence component values, hence frequency response curve.

$$x_1 = \frac{\omega_1}{\omega_o} \qquad a = \sqrt{\frac{2}{\frac{1}{x_1} - x_1 - 2}} \qquad b = a + \frac{1}{a}$$

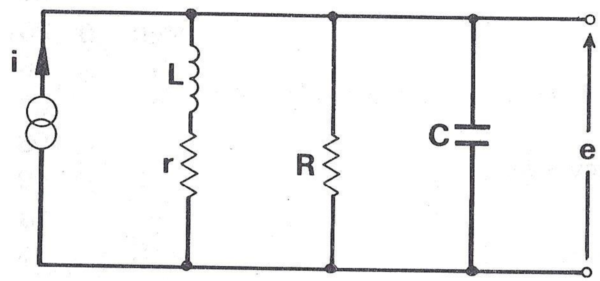
Execution:

x_1 / RUN / n / RUN / a / RUN / b

sto	2	00
÷	G	01
–	F	02
rcl	5	03
–	F	04
stop	0	05
#	3	06
2	2	07
÷	G	08
+	E	09
=	–	10
√x	1	11
stop	0	12
sto	2	13
÷	G	14
+	E	15
rcl	5	16
=	–	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

FILTERS

Single tuned circuit with losses



$$\omega_o = \frac{1}{\sqrt{LC}} \qquad R_o = \omega_o L = \frac{1}{\omega_o C} = \sqrt{\frac{L}{C}}$$

$$d_s = \frac{r}{\omega_o L} = \frac{r}{R_o} \qquad d_p = \frac{R_o}{R}$$

$$d = d_s + d_p \qquad Q = \frac{1}{d}$$

Normalised variables:

Normalised frequency $= x = \dfrac{\omega}{\omega_o}$

deviation $= v = Q\left(x - \dfrac{1}{x}\right)$

Normalised admittance:

$$y = YQR_o = \frac{1}{d}\left[d_p + \frac{d_s}{x^2 + d_s^2} + jx\left(1 - \frac{1}{x^2 + d_s^2}\right)\right]$$

Normalised impedance:

$$Z = \frac{1}{y} = \frac{e}{iQR_o} = \frac{Z}{QR_o} = d\left[d_p + \frac{d_s}{x^2 + d_s^2} + jx\left(1 - \frac{1}{x^2 + d_s^2}\right)\right]^{-1}$$

FILTERS

Single tuned circuit with losses (contd.)

For $Q \gg 1$, (or $Q > 10$), the frequency response is closely approximated by

$$\frac{e}{iR_o} = Q\,(1 + v^2)^{-1/2}$$

and can be found using the simple filters program.

For exact calculation, where $Q < 10$:

series resonant frequency $= \omega_o$

$x_o = 1$

in-phase resonant frequency $= \omega_r$

$x_r = \sqrt{1 - d_s^2}$

parallel resonant frequency $= \omega_p$

$x_p = \left[(1 + 2d_s d_p + 2d_s^2)^{1/2} - d_s^2\right]^{1/2}$

impedance at $\omega_r = R_r = QR_o$

Resonant frequencies

Execution:

d_s / RUN / x_r / d_p / RUN / x_p

sto	2	00
X	·	01
—	F	02
+	E	03
#	3	04
1	1	05
=	—	06
√x	1	07
stop	0	08
+	E	09
rcl	5	10
X	·	11
rcl	5	12
+	E	13
+	E	14
#	3	15
1	1	16
=	—	17
√x	1	18
—	F	19
(	6	20
rcl	5	21
X	·	22
)	6	23
=	—	24
√x	1	25
stop	0	26
▼	A	27
goto	2	28
0	0	29
0	0	30
		31
		32
		33
		34
		35

FILTERS

Single tuned circuit with losses (contd.)

Amplitude and phase response – Preliminary program

To find a and b:

$a = 2 + d_s^2 - d_p^2$

$b = 1 + 2d_p d_s + 2d_s^2$

Execution:

d_p / RUN / d_s / RUN / b / RUN / a

sto	2	00
X	·	01
—	F	02
+	E	03
(	6	04
stop	0	05
+	E	06
▼	A	07
MEx	5	08
X	·	09
rcl	5	10
+	E	11
+	E	12
#	3	13
1	1	14
=	—	15
stop	0	16
rcl	5	17
X	·	18
)	6	19
+	E	20
#	3	21
2	2	22
=	—	23
stop	0	24
▼	A	25
goto	2	26
0	0	27
0	0	28
		29
		30
		31
		32
		33
		34
		35

FILTERS

Single tuned circuit with losses (contd.)

Amplitude and phase response

$$|z| = d\left[u^2 - a + \frac{b}{u^2}\right]^{-1/2}$$

$$\phi = -\arctan \frac{x\,(u^2 - 1)}{u^2 d_p + d_s}$$

where $u^2 = x^2 + d_s^2$ $\qquad d = d_s + d_p$

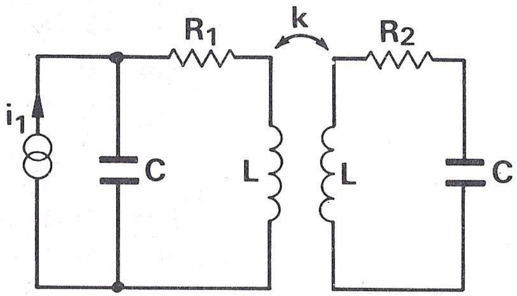
Execution:

x / RUN / d_s / RUN / b / RUN / a / RUN / d / |z| / X / iQR_o / = / e / d_p / RUN / d_s / RUN / X / RUN / ▲▼ / ▲▼ / arctan / ϕ

X	·	00
+	E	01
(	6	02
stop	0	03
X	·	04
)	6	05
+	E	06
sto	2	07
(	6	08
÷	G	09
X	·	10
stop	0	11
)	6	12
—	F	13
stop	0	14
÷	G	15
=	—	16
√x	1	17
X	·	18
stop	0	19
X	·	20
rcl	5	21
+	E	22
stop	0	23
÷	G	24
X	·	25
(	6	26
#	3	27
1	1	28
—	F	29
rcl	5	30
)	6	31
X	·	32
stop	0	33
=	—	34
stop	0	35

TUNED COUPLED CIRCUITS

Response of secondary circuit



Case of two tuned circuits having equal inductances and capacitances but unequal Q-factors

Normalised response in secondary (relative to output at ω_o when s = 1)

$$y_2 = \frac{2s}{1 + s^2 + jvb - v^2} \quad \text{where}$$

$$v = \sqrt{Q_1 Q_2}\left(x - \frac{1}{x}\right) \quad x = \frac{\omega}{\omega_o}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$b = \left(\frac{Q_1}{Q_2} + \frac{Q_2}{Q_1}\right) \qquad Q_1 = \frac{\omega_o L}{R_1} \qquad Q_2 = \frac{\omega_o L}{R_2}$$

$$s = k\sqrt{Q_1 Q_2}$$

$$k = \text{coupling factor} = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{L}$$

$$a = \sqrt{b + 2}$$

×	·	00
+	E	01
#	3	02
1	1	03
—	F	04
(	6	05
stop	0	06
×	·	07
)	6	08
=	—	09
sto	2	10
stop	0	11
—	F	12
×	·	13
stop	0	14
÷	G	15
rcl	5	16
×	·	17
(	6	18
▼	A	19
arctan	9	20
stop	0	21
rcl	5	22
)	6	23
×	·	24
+	E	25
(	6	26
rcl	5	27
×	·	28
)	6	29
=	—	30
√x	1	31
÷	G	32
+	E	33
×	·	34
stop	0	35

Magnitude:

$$|y_2| = \frac{2s}{[(1 + s^2 - v^2)^2 + b^2 v^2]^{½}} = \frac{2s}{\left[(1 + s^2)^2 - 2v^2\left(s^2 - \frac{b}{2}\right) + v^4\right]^{½}}$$

Phase:

$$\phi = -\arctan \frac{v\sqrt{b + 2}}{1 + s^2 - v^2}$$

Execution:

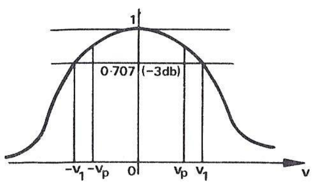
S / RUN / v / RUN / v / RUN / a / RUN / ϕ / RUN / s / = / $|y_2|$

Note: as |v| increases, ϕ changes sign. Correct value of ϕ when this happens is obtained by subtracting π if v is positive, adding π if v is negative.

To obtain ϕ in degrees, use / ▲▼ / ▲▼ / R→D / before final / RUN /. Correct sign change by subtracting 180°.

TUNED COUPLED CIRCUITS

Design for linear phase response



Theory:

$$\phi = -\arctan \frac{v\sqrt{b+2}}{1 + s^2 - v^2}$$

$$\frac{d\phi}{dv} = -\frac{\sqrt{b+2}\,(1 + s^2 + v^2)}{(1 + s^2) - 2v^2\left(s^2 - \frac{b}{2}\right) + v^4}$$

For maximally linear phase/frequency characteristic, the condition is:

$$s^2 = \frac{b-1}{3}$$

For maximum energy transfer the condition is $s = 1$ (critical coupling), hence to satisfy both conditions, $b = 4$ is optimum.

The frequency response is:

$$|y_2| = \frac{2s}{\left[\frac{(b+2)^2}{3} + v^2\left(\frac{b+2}{3}\right) + v^4\right]^{1/2}}$$

$$= \frac{2}{(4 + 2v^2 + v^4)^{1/2}} \quad \text{for } b = 4$$

+	E	00
#	3	01
2	2	02
÷	G	03
#	3	04
3	3	05
—	F	06
sto	2	07
#	3	08
1	1	09
=	—	10
$\sqrt{x}$	1	11
stop	0	12
÷	G	13
(	6	14
X	·	15
—	F	16
+	E	17
rcl	5	18
)	6	19
X	·	20
(	6	21
#	3	22
3	3	23
X	·	24
rcl	5	25
=	—	26
$\sqrt{x}$	1	27
)	6	28
=	—	29
▼	A	30
arctan	9	31
▼	A	32
goto	2	33
1	1	34
2	2	35

$$\phi_2 = -\arctan \frac{v\sqrt{b+2}}{\frac{b+2}{3} - v^2} = -\arctan \frac{v\sqrt{6}}{2 - v^2}$$

Program computes s and ϕ_2 given b.

v_1 can be obtained by post-execution sequence.

Execution:

b / RUN / s / v / RUN / ϕ_2 (repeat for any other values of v)
/ v / RUN / ϕ_2 · · ·

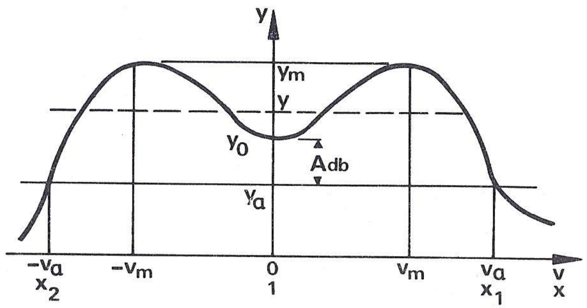
Bandwidth to 1% deviation from phase linearity: $v_p = \cdot 49601\sqrt{1 + s^2}$

$\phi_2 = 1{\cdot}1394443$ for 1% deviation from phase linearity.

Attenuation at $v_p = -1{\cdot}1608$ dB relative to centre frequency.

TUNED COUPLED CIRCUITS –

Bandwidth to given attenuation



Let $\alpha = \frac{y_\alpha}{y_o}$, the attenuation at v_α

relative to that at v = 0.

Then

$$v_\alpha^2 = \left(s^2 - \frac{b}{2}\right) \pm \sqrt{\left(s^2 - \frac{b}{2}\right)^2 + (1+s^2)^2\left(\frac{1}{\alpha^2} - 1\right)}$$

The + sign gives values outside the peaks.

The – sign gives values inside the peaks,

but only for $s^2 > \frac{b}{2}$ and $\alpha > 1$ (see dashed line).

If $y_\alpha > y_m$ or these conditions are not observed an error will be indicated.

$$v_m^2 = s^2 - \frac{b}{2}$$

X	·	00
÷	G	01
–	F	02
#	3	03
1	1	04
X	·	05
(	6	06
stop	0	07
X	·	08
+	E	09
sto	2	10
#	3	11
1	1	12
X	·	13
)	6	14
+	E	15
(	6	16
stop	0	17
–	F	18
÷	G	19
#	3	20
2	2	21
+	E	22
rcl	5	23
X	·	24
sto	2	25
)	6	26
=	–	27
$\sqrt{x}$	1	28
▼	A	29
MEx	5	30
stop	0	31
rcl	5	32
=	–	33
$\sqrt{x}$	1	34
stop	0	35

To find α from A dB:

A / – / ÷ / 8·68589 / = / ▲▼ / ▲▼ / e^x / α

Execution:

α / RUN / s / RUN / b / RUN / + / RUN / v_α outside peaks

α / RUN / s / RUN / b / RUN / – / RUN / v_α inside peaks

Error symbols:

If an error symbol occurs after / b / RUN / but before entering + or –, the value of α entered is too large (< ratio of peak to valley).

If an error symbol occurs after / d / – / RUN /, either

$s^2 \not> \frac{b}{2}$ or $\alpha < 1$.

Post execution:

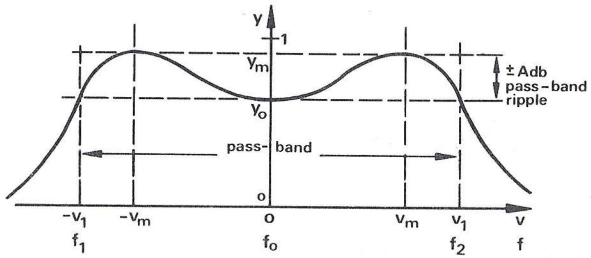
To find x from v:

v / ÷ / Q / X / ▲▼ / sto / – / 1 / = / ▲▼ / $\sqrt{x}$ / + / ▲▼ / rcl / = / x_1 / ÷ / = / x_2

(multiply x_1 or x_2 by f_o to obtain f_1 or f_2)

TUNED COUPLED CIRCUITS

Design for given bandwidth and pass-band ripple



Peak to valley ratio:

$$a = 10^{0 \cdot 1A} = e^{\frac{A}{4 \cdot 34294}}$$

$$a = \frac{y_m}{y_o} = \frac{1 + s^2}{\left(1 + s^2(b+2) - \frac{b^2}{4}\right)^{½}}$$

where $s = k\sqrt{Q_1 Q_2}$, $b = \frac{Q_1}{Q_2} + \frac{Q_2}{Q_1}$

∴ coupling for given peak to valley ratio:

$$s^2 = \frac{\frac{b}{2} + \sqrt{1 - a^{-2}}}{1 - \sqrt{1 - a^{-2}}}$$

Location of peaks:

$$v_m = \sqrt{s^2 - \frac{b}{2}}$$

Location of pass-band edges:

$$v_1 = \sqrt{2s^2 - b} = \sqrt{2}\, v_m$$

X	·	00
÷	G	01
—	F	02
+	E	03
#	3	04
1	1	05
=	—	06
√x	1	07
sto	2	08
—	F	09
+	E	10
#	3	11
1	1	12
÷	G	13
(	6	14
stop	0	15
÷	G	16
#	3	17
2	2	18
+	E	19
▼	A	20
MEx	5	21
)	6	22
÷	G	23
—	F	24
▼	A	25
MEx	5	26
+	E	27
=	—	28
√x	1	29
stop	0	30
▼	A	31
MEx	5	32
√x	1	33
stop	0	34
=	—	35

Relation of Q to v, and band width:

$$Q = \sqrt{Q_1 Q_2} = \frac{v_1 f_o}{f_2 - f_1}$$

$$x = \frac{\omega}{\omega_o} = \frac{f}{f_o}$$

$$v = Q\left(x - \frac{1}{x}\right)$$

f_2 = upper limit of pass-band
f_1 = lower limit of pass-band
f_o = centre frequency $= \sqrt{f_1 f_2}$

To find a from A:

A / ÷ / 4·34294 / = / ▲▼ / ▲▼ / e^x / a

Execution:

Either:

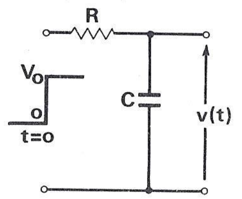
a / RUN / b / RUN / v_1 / X / f_o / ÷ / ▲▼ / (/ f_2 / — / f_1 / ▲▼ /) / = / Q / RUN / s / ÷ / ▲▼ / rcl / = / k

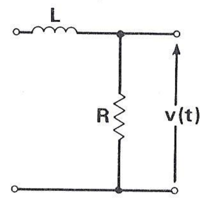
Or:

a / RUN / b / RUN / v_1 / RUN / s

LINEAR CIRCUIT THEORY

Simple L–R or C–R circuit





$\tau = CR$ or $\tau = \frac{L}{R}$

Charge: $V_c(t) = V_o(1 - e^{-\frac{t}{\tau}})$

Discharge: $V_d(t) = V_o e^{-\frac{t}{\tau}}$

Pre-execution:

R / X / C / = / ▲▼ / sto / *or*
L / ÷ / R / = / ▲▼ / sto / *or*
τ / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

t / RUN / V_o / RUN / $V_d(t)$

÷	G	00
rcl	5	01
–	F	02
=	–	03
▼	A	04
e^x	4	05
X	·	06
stop	0	07
=	–	08
stop	0	09
▼	A	10
goto	2	11
0	0	12
0	0	13
		14
		15
		16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

LINEAR CIRCUIT THEORY

Simple L–R or C–R circuit (contd.)

Pre-execution:

R / X / C / = / ▲▼ / sto / *or*
L / ÷ / R / = / ▲▼ / sto / *or*
τ / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

t / RUN / V_o / RUN / $V_c(t)$

÷	G	00
rcl	5	01
–	F	02
=	–	03
▼	A	04
e^x	4	05
–	F	06
+	E	07
#	3	08
1	1	09
X	·	10
stop	0	11
=	–	12
stop	0	13
▼	A	14
goto	2	15
0	0	16
0	0	17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

LINEAR CIRCUIT THEORY

Simple L–R or C–R circuit (contd.)

÷	G	00
rcl	5	01
–	F	02
=	–	03
▼	A	04
e^x	4	05
X	·	06
stop	0	07
–	F	08
stop	0	09
–	F	10
=	–	11
stop	0	12
▼	A	13
goto	2	14
0	0	15
0	0	16
		17
		18
		19
		20
		21
		22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

Pre-execution:

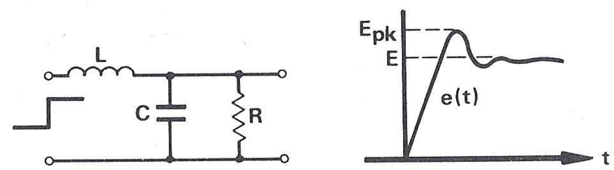
R / X / C / = / ▲▼ / sto / *or*
L / ÷ / R / = / ▲▼ / sto / *or*
τ /▲▼/ sto / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

t / RUN / V_o / RUN / $V_d(t)$ / V_o / RUN / $V_c(t)$

LINEAR CIRCUIT THEORY

Damping factor from transient response



overshoot (y) = $\left(\frac{E_{pk}}{E} - 1\right)$ $\quad 0 \leqslant y \leqslant 1$

$$K = \frac{X}{\sqrt{\pi^2 + X^2}} \quad \text{where } X = -\ln\left(\frac{E_{pk}}{E} - 1\right)$$

Note: This formula applies to ideal 2nd-order systems of all kinds.

Pre-execution:

To enter first set of values
▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

E_{pk} / RUN / E / RUN / y / RUN / k
E'_{pk} / RUN / y′ / RUN / k′
(continue for other values of E_{pk} at same E)

–	F	00
stop	0	01
sto	2	02
÷	G	03
rcl	5	04
=	–	05
stop	0	06
ln	4	07
–	F	08
÷	G	09
#	3	10
3	3	11
·	A	12
1	1	13
4	4	14
1	1	15
5	5	16
9	9	17
3	3	18
÷	G	19
(	6	20
X	·	21
+	E	22
#	3	23
1	1	24
=	–	25
$\sqrt{x}$	1	26
)	6	27
=	–	28
stop	0	29
–	F	30
rcl	5	31
▼	A	32
goto	2	33
0	0	34
3	3	35

LINEAR CIRCUIT THEORY

Time taken to reach given voltage

÷	G	00
stop	0	01
—	F	02
(	6	03
ln	4	04
X	·	05
rcl	5	06
=	—	07
stop	0	08
#	3	09
1	1	10
=	—	11
)	6	12
—	F	13
=	—	14
ln	4	15
X	·	16
rcl	5	17
=	—	18
stop	0	19
▼	A	20
goto	2	21
0	0	22
0	0	23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

$$t_d = -\tau \ln \frac{v_d(t)}{V_o}, \quad t_c = -\tau \ln\left(1 - \frac{v_c(t)}{V_o}\right)$$

Pre-execution:

$-\tau$ / ▲▼ / sto / *or*

L / + / R / = / ▲▼ / sto / *or*

τ / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

v(t) / RUN / V_o / RUN / t_d / RUN / t_c

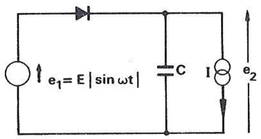
Special case:

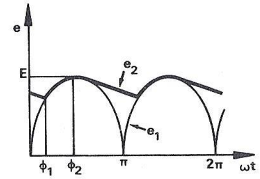
Rise-time —

Compute for $v(t) = 0{\cdot}1V_o$ $\quad t_r = t_d - t_c = 2{\cdot}19714\tau$

FULL—WAVE RECTIFIER WITH CAPACITOR SMOOTHING

sto	2	00
▼	A	01
arccos	8	02
X	·	03
▼	A	04
MEx	5	05
—	F	06
+	E	07
(	6	08
rcl	5	09
sin	7	10
)	6	11
=	—	12
stop	0	13
sto	2	14
cos	8	15
X	·	16
rcl	5	17
—	F	18
(	6	19
rcl	5	20
sin	7	21
)	6	22
+	E	23
stop	0	24
÷	G	25
(	6	26
rcl	5	27
cos	8	28
+	E	29
stop	0	30
▼	A	31
goto	2	32
1	1	33
1	1	34
		35





The diode conducts from ϕ_1 to ϕ_2 in each input cycle where

$$\cos\phi_2 = -\frac{I}{\omega CE} = -x$$

$$\sin\phi_1 + x\,\phi_1 = \sin(\arccos x) - x \arccos x = k$$

This program finds ϕ_2 and then calculates ϕ_1 using the Newton-Raphson iterative formula

$$\phi_1' = \frac{\phi_1 \cos\phi_1 - \sin\phi_1 + k}{\cos\phi_1 + x}$$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 /

Execution:

x / RUN / k

3·14159 / — / ▲▼ / rcl / = / ϕ_2

/ ▲▼ / rcl / $\pi - \phi_2$ (used as starting value ϕ_1)

ϕ_1 / RUN / k / RUN / x / RUN / ϕ_1'

repeat until convergence obtained.
(ϕ_1 is also in memory)

Given ϕ_1 and ϕ_2 all the useful circuit parameters can be calculated. (see over)

RECTIFIER WITH CAPACITIVE SMOOTHING

Ripple voltage:

$v_{r\ pk\text{-}pk} = E\ (1 - \sin \phi_1)$

Post execution:

▲▼ / sin / — / + / 1 / X / E / = / $v_{r\ pk\text{-}pk}$

Peak rectifier current:

$i_{d\ pk} = I + \omega CE \cos \phi_1 = I \left(1 + \frac{\cos \phi_1}{x}\right)$

Post execution:

▲▼ / rcl / ▲▼ / cos / ÷ / x / + / 1 / X / I / = / $i_{d\,pk}$

RECTIFIER WITH CAPACITIVE SMOOTHING

Calculate ϕ_1 and ϕ_2 using program given (page 45).

Mean rectified voltage:

$$\overline{e_2} = \frac{2}{\pi} E \sin a\ (\cos b' + b' \sin b')$$

$$\text{when } a = \frac{\phi_1 + \phi_2}{2}, \quad b' = \frac{\phi_1 + \pi - \phi_2}{2}$$

$$b = \frac{\phi_1 - \phi_2}{2}$$

Pre-execution:

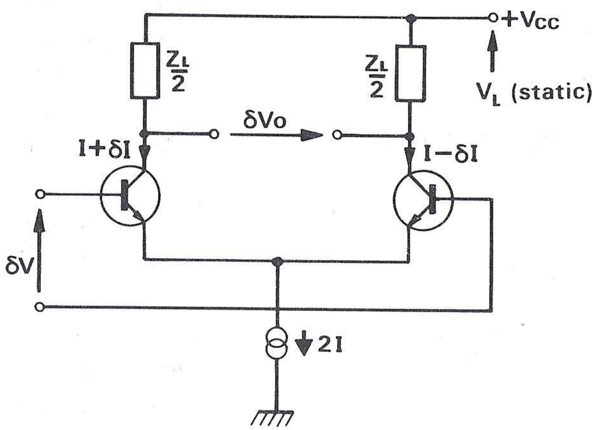
ϕ_1 / + / ϕ_2 / ▲▼ / sto / ÷ / 2 / — / a / ▲▼ / ▲▼ / MEx / + / b

Execution:

/ RUN / E / = / $\overline{e_2}$

(	6	00
#	3	01
1	1	02
·	A	03
5	5	04
7	7	05
0	0	06
8	8	07
=	—	08
▼	A	09
MEx	5	10
sin	7	11
÷	G	12
rcl	5	13
=	—	14
▼	A	15
MEx	5	16
)	6	17
=	—	18
▼	A	19
MEx	5	20
X	·	21
(	6	22
rcl	5	23
sin	7	24
X	·	25
rcl	5	26
=	—	27
▼	A	28
MEx	5	29
cos	8	30
+	E	31
rcl	5	32
)	6	33
X	·	34
stop	0	35

TRANSFER FUNCTION OF LONG–TAILED PAIR



$$\delta V = \frac{KT}{q}\,\ell n\left(\frac{1+\frac{\delta I}{I}}{1-\frac{\delta I}{I}}\right)$$

$$\frac{\delta I}{I} = \frac{\exp\left(\frac{q\delta V}{kT}\right) - 1}{\exp\left(\frac{q\delta V}{kT}\right) + 1}$$

$\delta V_o = Z_L\,\delta I$

q = electronic charge = $1{\cdot}602192 \times 10^{-19}$ C

k = Boltzmann's constant = $1{\cdot}380622 \times 10^{-23}\ JK^{-1}$

T = absolute temperature (°C + 273·15)

$V_L = \frac{IR_L}{2}$ (if load is resistive)

X	·	00
#	3	01
8	8	02
·	A	03
6	6	04
1	1	05
7	7	06
1	1	07
·	A	08
·	A	09
5	5	10
=	—	11
sto	2	12
stop	0	13
÷	G	14
rcl	5	15
=	—	16
▼	A	17
e^x	4	18
—	F	19
#	3	20
1	1	21
÷	G	22
(	6	23
+	E	24
#	3	25
2	2	26
=	—	27
)	6	28
X	·	29
stop	0	30
=	—	31
▼	A	32
goto	2	33
1	1	34
3	3	35

(set temperature:)

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / T / RUN

Execution:

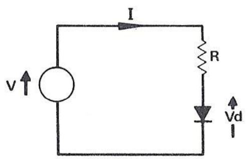
δV / RUN / $\frac{\delta I}{I}$ { I / RUN / δI ; I / X / Z_L / RUN / δV_o ; V_L / + / RUN / δV_o }

Repeat for all required values of δV

e.g. for sine wave, δV = V sin ωt,

/ ω / X / t / = /▲▼/ sin / X / V / RUN / I / RUN / δI etc.

OPERATING POINT OF DIODE–RESISTOR COMBINATION



$$V = IR + \frac{nkT}{q}\,\ell n\left(1 + \frac{I}{I_s}\right)$$

Newton-Raphson method gives the iteration formula for I

$$I' = \frac{V + \frac{nkT}{q}\left(\frac{I}{I + I_s}\right) - \frac{nkT}{q}\,\ell n\left(1 + \frac{I}{I_s}\right)}{R + \frac{nkT}{q}\left(\frac{I}{I + I_s}\right)}$$

For forward-biased diodes, $I \gg I_s$, so this simplifies to

$$I' \doteq \frac{V + \frac{nkT}{q}\left(1 - \ell n\frac{I}{I_s}\right)}{R + \frac{nkT}{qI}}$$

If I is mA and V_o = diode voltage at I_o = 1mA,

$$I' \doteq \frac{V - V_o + \frac{nkT}{q}\left(1 - \ell n\frac{I}{I_o}\right)}{R + \frac{nkT}{qI}}$$

where $V_o = \frac{nkT}{q}\,\ell n\frac{I_o}{I_s}$

÷	G	00
X	·	01
(	6	02
ln	4	03
sto	2	04
#	3	05
·	A	06
0	0	07
8	8	08
6	6	09
1	1	10
7	7	11
1	1	12
X	·	13
stop	0	14
X	·	15
▼	A	16
MEx	5	17
+	E	18
rcl	5	19
+	E	20
stop	0	21
=	–	22
▼	A	23
MEx	5	24
)	6	25
+	E	26
stop	0	27
÷	G	28
rcl	5	29
÷	G	30
=	–	31
=	–	32
=	–	33
=	–	34
stop	0	35

Consistent units are:

V in mV, R in Ω, I in mA

n = 1 for germanium diodes or for transistor junctions

n = 1·5 for silicon p-n diodes

Find $\frac{nkT}{q}$ to use in program (in mV)

or use T each time in program execution if desired.

Execution:

(with T) ▲▼ / ▲▼ / goto / 0 / 0 /

$$I\ /\ RUN\ /\left\{\begin{array}{l}T\ /\ RUN\ /\\ T\ /\ X\ /\ n\ /\ RUN\ /\end{array}\right\}\left\{\begin{array}{l}V\ /\ -\ /\ V_o\\ V - V_o\end{array}\right\}/\ RUN\ /\ R\ /\ RUN\ /\ I'$$

$$/RUN\ /\left\{\begin{array}{l}T\\ T\ /\ X\ /\ n\end{array}\right\}/\ RUN\ /\left\{\begin{array}{l}V\ /\ -\ /\ V_o\\ V - V_o\end{array}\right\}/\ RUN\ /\ R\ /\ RUN\ /\ I''$$

(repeat until values converge)

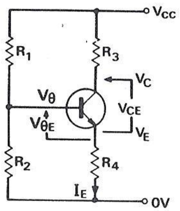
(without T) – enter a constant as indicated (to lesser accuracy as required) at steps 06 to 14

$$I\ /\ RUN\ /\left\{\begin{array}{l}V - V_o\\ V\ /\ -\ /\ V_o\end{array}\right\}/\ RUN\ /\ R\ /\ RUN\ /\ I'$$

$$/\ RUN\ /\left\{\begin{array}{l}V - V_o\\ V\ /\ -\ /\ V_o\end{array}\right\}/\ RUN\ /\ R\ /\ RUN\ /\ I''$$

($\frac{nkT}{q}$ may be found from: / n / X / T / X / 1·086171 / = / $\frac{nkT}{q}$ mV;
at 25°C $\frac{kT}{q} \doteq 25{\cdot}6789$ mV)

OPERATING POINT OF TRANSISTOR IN BASE–POTENTIAL DIVIDER AND EMITTER RESISTOR BIAS



Preliminary equations:

$$V = \frac{V_{CC} R_2}{R_1 + R_2}$$

$$R = R_4 + \frac{R_1 R_2}{(R_1 + R_2)\ (h_{FE} + 1)}$$

I_E is given by the diode-resistor program with $V_o = V_{BE}$ of transistor at 1 mA, R and V as given above, and n = 1.

Circuit equations:

$$V_E = I_E R_4 \qquad I_C = I_E \frac{h_{FE}}{1 + h_{FE}}$$

$$V_{BE} = \frac{k}{q} \ell n\, I_E (mA) + V_o$$

$$V_B = V_E + V_{BE}$$

$$V_C = V_{CC} - I_E R_3 \frac{h_{FE}}{1 + h_{FE}}$$

$$V_{CE} = V_C - V_E$$

Prelim. program

+	E	00
stop	0	01
sto	2	02
÷	G	03
(	6	04
—	F	05
rcl	5	06
)	6	07
÷	G	08
X	·	09
▼	A	10
MEx	5	11
÷	G	12
stop	0	13
+	E	14
stop	0	15
=	—	16
stop	0	17
X	·	18
rcl	5	19
—	F	20
stop	0	21
=	—	22
stop	0	23
▼	A	24
goto	2	25
0	0	26
0	0	27
		28
		29
		30
		31
		32
		33
		34
		35

1. Enter preliminary program . . .

Execution:

R_2 / RUN / R_1 / RUN / h_{FE} + 1 / RUN / R_4 / RUN / R

V_{CC} / RUN / V / V_o / RUN / V — V_o

2. Next enter diode and resistor program (see page 50) and execute to find I_E

3. Finally enter program in box and run:

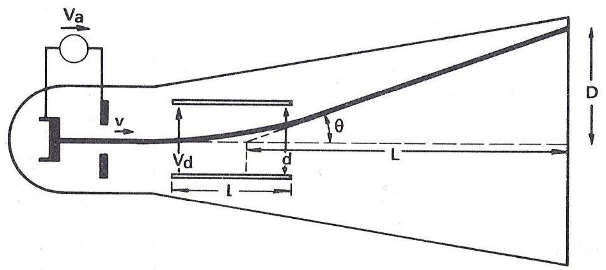
Execution:

I / RUN / T / RUN / V_o / RUN / V_{BE} / R_4 / RUN / V_E / RUN / V_B / h_{FE} / RUN / $-I_C$ / R_3 / + / V_{CC} / — / V_C / V_E / = / V_{CE}

Final program

sto	2	00
ln	4	01
X	·	02
#	3	03
·	A	04
0	0	05
8	8	06
6	6	07
1	1	08
7	7	09
1	1	10
X	·	11
stop	0	12
+	E	13
stop	0	14
+	E	15
(	6	16
stop	0	17
X	·	18
rcl	5	19
)	6	20
stop	0	21
=	—	22
stop	0	23
÷	G	24
(	6	25
+	E	26
#	3	27
1	1	28
=	—	29
)	6	30
—	F	31
X	·	32
rcl	5	33
X	·	34
stop	0	35

ELECTRON DYNAMICS



(S.I. Units)

To find electrostatic deflection, velocity, sensitivity, deflection and angle of deflection in cathode ray tube.
(non-relativistic)

$$v = \sqrt{\frac{2eV_a}{m}}$$

$$S = \frac{lL}{2dV_a}$$

$$D = \frac{lLV_d}{2dV_a} = SV_d$$

$$\theta = \arctan\frac{D}{L} = \arctan\frac{lV_d}{2dV_a}$$

e = electron charge = $1{\cdot}6022 \times 10^{-19}$ C

m = electron mass = $9{\cdot}1096 \times 10^{-31}$ kg

Execution:

V_a / RUN / v / d / RUN / l / RUN / L / RUN / S / V_d / RUN / D / RUN / θ

sto	2	00
√x	1	01
X	·	02
#	3	03
5	5	04
·	A	05
9	9	06
3	3	07
0	0	08
9	9	09
·	A	10
5	5	11
=	—	12
stop	0	13
+	E	14
÷	G	15
X	·	16
stop	0	17
÷	G	18
rcl	5	19
X	·	20
stop	0	21
sto	2	22
X	·	23
stop	0	24
÷	G	25
stop	0	26
rcl	5	27
=	—	28
▼	A	29
arctan	9	30
stop	0	31
▼	A	32
goto	2	33
0	0	34
0	0	35

DEFLECTION OF RELATIVISTIC ELECTRONS

Small transverse field as in cathode ray tube

$$\frac{D}{L} = \tan\theta \triangleq \frac{eV_d}{mc^2}\,\frac{l}{d} \times \left[\left(1 + \frac{eV_a}{mc^2}\right) - \left(1 + \frac{eV_a}{mc^2}\right)^{-1}\right]^{-1}$$

Execution:

for D or θ only –

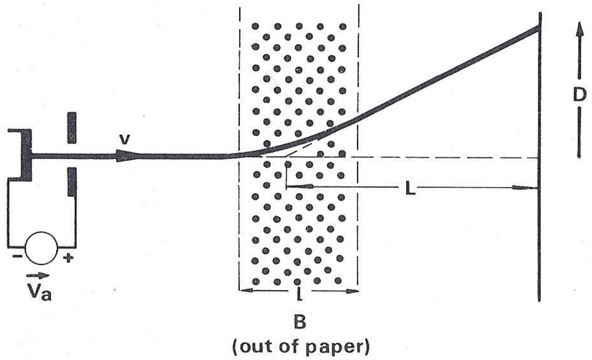
V_a / RUN / V_d / RUN / d / RUN / l / RUN /
tan θ { / X / L / = / D
/ RUN / θ

or, for S, D and θ

V_a / RUN / L / RUN / d / RUN / l / RUN / S / X / V_d / ÷ / D / L / = / tan θ / RUN / θ

X	·	00
(	6	01
#	3	02
1	1	03
·	A	04
9	9	05
5	5	06
6	6	07
9	9	08
·	A	09
·	A	10
6	6	11
=	—	12
sto	2	13
)	6	14
+	E	15
#	3	16
1	1	17
—	F	18
(	6	19
÷	G	20
)	6	21
÷	G	22
X	·	23
rcl	5	24
X	·	25
stop	0	26
÷	G	27
stop	0	28
X	·	29
stop	0	30
=	—	31
stop	0	32
▼	A	33
arctan	9	34
stop	0	35

MAGNETIC DEFLECTION IN CATHODE–RAY TUBE (non-relativistic)



$$\theta = \arcsin \frac{leB}{mv} = \arcsin \frac{lB}{\sqrt{V_a}} \sqrt{\frac{e}{2m}}$$

$$D = L \tan \theta$$

$$S = \frac{D}{B} \triangleq \frac{lL}{\sqrt{V_a}} \sqrt{\frac{e}{2m}}$$ (magnetic deflection sensitivity for small θ)

Execution:

V / RUN / l / RUN / B / RUN / θ / RUN / L / RUN / S / RUN / D

Notes:

1. In practical wide angle tubes the field will not be uniform.
2. If $\theta > \frac{\pi}{2}$ is computed, a value of 0 with no error symbol will be shown. This means the electron is reversed in direction by the field.

√x	1	00
÷	G	01
X	·	02
#	3	03
2	2	04
9	9	05
6	6	06
5	5	07
4	4	08
6	6	09
X	·	10
stop	0	11
X	·	12
sto	2	13
stop	0	14
=	—	15
▼	A	16
arcsin	7	17
stop	0	18
tan	9	19
X	·	20
(	6	21
stop	0	22
X	·	23
▼	A	24
MEx	5	25
=	—	26
stop	0	27
rcl	5	28
)	6	29
=	—	30
stop	0	31
▼	A	32
goto	2	33
0	0	34
0	0	35

VELOCITY OF ACCELERATED ION (non-relativistic)

M = mass of ion
ne = charge on ion
V = accelerating potential (volts)

$$v = \sqrt{\frac{2neV}{M}}$$

Execution:

V / RUN / n / RUN / M / RUN / v

X	·	00
#	3	01
3	3	02
·	A	03
2	2	04
0	0	05
4	4	06
4	4	07
·	A	08
·	A	09
1	1	10
9	9	11
X	·	12
stop	0	13
÷	G	14
stop	0	15
=	—	16
√x	1	17
stop	0	18
▼	A	19
goto	2	20
0	0	21
0	0	22
		23
		24
		25
		26
		27
		28
		29
		30
		31
		32
		33
		34
		35

MASS AND VELOCITY OF ACCELERATED ELECTRON OR ION (relativistic)

V = accelerating potential (volts)

$$m_r = m\left(1 + \frac{eV}{mc^2}\right)$$

$$v_r = c\sqrt{1 - \left(1 + \frac{eV}{mc^2}\right)^{-2}}$$

For electron

$e = 1{\cdot}6022 \times 10^{-19}$ C

$m = 9{\cdot}1096 \times 10^{-31}$ kg

$c = 2{\cdot}9979 \times 10^{8}$ ms^{-1}

$$\frac{e}{mc^2} = 1{\cdot}9569 \times 10^{-6}\ V^{-1}$$

Execution:

V / RUN / v_r / ▲▼ / rcl / X / 9·1096 / · / · / 31 / = / m_r

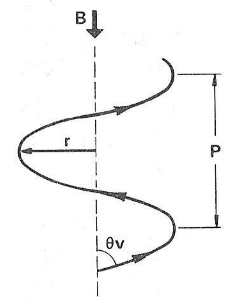
For ion of mass M and charge ne:

n / X / V / X / m / ÷ / M / RUN / v_r / ▲▼ / rcl / X / M / = / M_r

X	·	00
#	3	01
1	1	02
·	A	03
9	9	04
5	5	05
6	6	06
9	9	07
·	A	08
·	A	09
6	6	10
+	E	11
#	3	12
1	1	13
=	—	14
sto	2	15
÷	G	16
X	·	17
—	F	18
+	E	19
#	3	20
1	1	21
=	—	22
$\sqrt{x}$	1	23
X	·	24
#	3	25
2	2	26
·	A	27
9	9	28
9	9	29
7	7	30
9	9	31
·	A	32
8	8	33
=	—	34
stop	0	35

ELECTRON MOTION IN TRANSVERSE MAGNETIC FIELD

Radius and period of orbit, pitch of helical path.



Period $T = \frac{2\pi m}{eB}$ radius of circular path $r_c = \frac{vT}{2\pi}$

Radius of path r =

$$\frac{mv}{eB}\sin\theta = \frac{\sqrt{2m}}{e}\frac{\sqrt{V}}{B}\sin\theta = \frac{vT}{2\pi}\sin\theta$$

Pitch of path P =

$$\frac{2\pi mv}{eB}\cos\theta = 2\pi\frac{\sqrt{2m}}{e}\frac{\sqrt{V}}{B}\cos\theta = vT\cos\theta$$

θ = angle of injection (relative to B)

$$\left(\frac{2\pi m}{e} \doteq 3{\cdot}5724 \times 10^{-11}\right)$$

Pre-execution (if desired):

V / ▲▼ / $\sqrt{x}$ / X / 5·9309·5 / = / v

Execution:

v / RUN / B / = / T / RUN / r_c / θ / RUN / r / θ / RUN / = / P

X	·	00
(	6	01
#	3	02
3	3	03
·	A	04
5	5	05
7	7	06
2	2	07
4	4	08
·	A	09
·	A	10
1	1	11
1	1	12
÷	G	13
stop	0	14
)	6	15
÷	G	16
sto	2	17
#	3	18
6	6	19
·	A	20
2	2	21
8	8	22
3	3	23
2	2	24
X	·	25
(	6	26
stop	0	27
sin	7	28
)	6	29
=	—	30
stop	0	31
cos	8	32
X	·	33
rcl	5	34
stop	0	35

CAPACITANCE OF SPHERE, CONCENTRIC SPHERES, CONCENTRIC CYLINDERS

(i) Sphere of radius a:

$C = 4\pi\epsilon_o \epsilon_r a$

(ii) Concentric spheres of radii a and b (b > a)

$$C = 4\pi\epsilon_o \epsilon_r \frac{ab}{b - a}$$

(iii) Concentric cylinders of radii a and b (b > a), and length L:

$$C = \frac{4\pi\, \epsilon_o \epsilon_r L}{2 \ln\left(\frac{b}{a}\right)}$$

Pre-execution and execution:

(i) Sphere:

▲▼ / ▲▼ / goto / 1 / 9 / a / RUN / ϵ_r / RUN / C

(ii) Concentric spheres:

▲▼ / ▲▼ / goto / 1 / 2 / a / RUN / b / RUN / ϵ_r / RUN / C

(iii) Concentric cylinders:

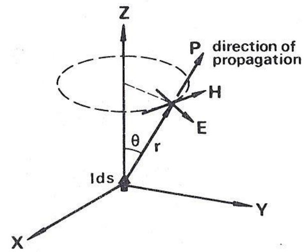
▲▼ / ▲▼ / goto / 0 / 0 / b / RUN / a / RUN / L / RUN / ϵ_r / RUN / C

($4\pi\epsilon_o = 1{\cdot}11265 \times 10^{-10}\ \mathrm{F\,m^{-1}}$)

(S.I. units)

÷	G	00
stop	0	01
=	–	02
ln	4	03
+	E	04
÷	G	05
X	·	06
stop	0	07
▼	A	08
goto	2	09
1	1	10
9	9	11
÷	G	12
–	F	13
(	6	14
stop	0	15
÷	G	16
)	6	17
÷	G	18
X	·	19
#	3	20
1	1	21
·	A	22
1	1	23
1	1	24
2	2	25
6	6	26
5	5	27
·	A	28
·	A	29
1	1	30
0	0	31
X	·	32
stop	0	33
=	–	34
stop	0	35

FIELD STRENGTH AND POYNTING VECTOR DUE TO ELECTRIC DIPOLE



$$H = \frac{Ids}{2\lambda r} \sin\theta \sin\left(\omega t - \frac{2\pi r}{\lambda}\right)$$

$E = Z_i H$ where $Z_i = \sqrt{\dfrac{\mu_o}{\epsilon_o}} = \mu_o c \triangleq 376{\cdot}73\Omega$

$P = EH$ (power flow per unit area)

$$P_{av} = \frac{E_{pk} H_{pk}}{2}$$

$\lambda = \dfrac{c}{f}$ where $c = 2{\cdot}9979 \times 10^8\ \mathrm{ms^{-1}}$

Execution:

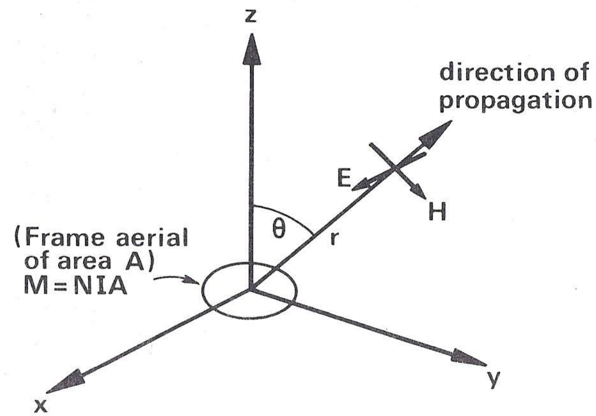
/ ▲▼ / ▲▼ / goto / 0 / 0 / f / RUN / λ
or / ▲▼ / ▲▼ / goto / 1 / 3 / λ

/ RUN / θ / RUN / r / RUN / { Ids or I / X / ds }

/ RUN / H_{pk} / RUN / E_{pk} / X / ▲▼ / rcl / ÷ / P_{pk} / 2 / = / P_{av}

÷	G	00
#	3	01
2	2	02
·	A	03
9	9	04
9	9	05
7	7	06
9	9	07
·	A	08
8	8	09
÷	G	10
=	–	11
stop	0	12
+	E	13
÷	G	14
X	·	15
(	6	16
stop	0	17
sin	7	18
)	6	19
÷	G	20
stop	0	21
X	·	22
stop	0	23
X	·	24
stop	0	25
sto	2	26
#	3	27
3	3	28
7	7	29
6	6	30
·	A	31
7	7	32
3	3	33
=	–	34
stop	0	35

RADIATION FROM LOOP (OR FERRITE) ANTENNA



$$H = NIA \frac{\pi}{\lambda^2 r} \sin\theta \sin\left(\omega t - \frac{2\pi r}{\lambda}\right)$$

$$E = Z_i H$$

$$P = EH$$

$$P_{av} = \frac{E_{pk} H_{pk}}{2}$$

For ferrite, replace NIA by NIA μ_{eff}

Additional formulae:

Radiation resistance:

$$R_r = \frac{16\pi^3}{3} Z_i \left(\frac{NA}{\lambda^2}\right)^2 = 62298{\cdot}7 \left(\frac{NA}{\lambda^2}\right)^2$$

Total power radiated:

$$P_r = I_{rms}^2 R_r = \frac{V_{rms}^2}{R_r} = \frac{I^2 R_r}{2}$$

X	·	00
÷	G	01
X	·	02
stop	0	03
X	·	04
sto	2	05
#	3	06
3	3	07
·	A	08
1	1	09
4	4	10
1	1	11
6	6	12
X	·	13
(	6	14
stop	0	15
sin	7	16
)	6	17
X	·	18
stop	0	19
X	·	20
stop	0	21
=	–	22
stop	0	23
rcl	5	24
X	·	25
X	·	26
#	3	27
6	6	28
2	2	29
2	2	30
9	9	31
9	9	32
=	–	33
=	–	34
stop	0	35

Execution:

λ / RUN / {
NA
N / X / A
N / X / 3·14159 / X / ▲▼ / (/ R / X / ▲▼ /)
N / X / ℓ / X / b
etc. }

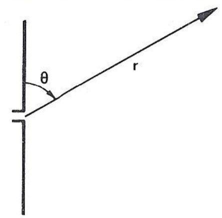
(/ X / μ_{eff})* / RUN / θ / RUN / r / RUN / I / RUN / H_{pk}

{ / X / 376·73 / = / E_{pk}
/ X / X / 377 / = / P_{pk}
/ X / X / 188·365 / = / P_{av} } / RUN / R_r

* omit these two terms for air-cored loop.

Note: Not applicable to near-field radiation pattern, $r < 10R$ where R = radius of loop.

RADIATION FROM HALF–WAVE DIPOLE



$$H = \frac{I}{2\pi r}\;\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\;\sin\left(\omega t - \frac{2\pi r}{\lambda}\right)$$

$E = Z_i H$

$P = HE \qquad P_{av} = \frac{H_{pk}E_{pk}}{2} \qquad Z_i \triangleq 377\Omega$

Additional formulae:

Radiation resistance:

$$R_r = \frac{\mu_o c}{4}\left(\ln 2\pi y + \int_{2\pi}^{\infty}\frac{\cos y}{y}\,dy\right) \triangleq 72{\cdot}9\Omega$$

Power outputs:

$$P_r = \frac{V_{rms}}{R_r} = I^2_{rms}R_r = \frac{I^2 R_r}{2}$$

(since I = peak current)

Execution:

θ / RUN / r / RUN / I / X / H_{pk} /

{RUN / E_{pk}
{X / RUN / P_{pk} / ÷ / 2 / = / P_{av}

This also applies to ¼-wave unipole above ground (radiation resistance 36·5Ω)

Range $0{\cdot}16 < \theta \leqslant 1{\cdot}57$

sto	2	00
cos	8	01
X	·	02
#	3	03
1	1	04
·	A	05
5	5	06
7	7	07
0	0	08
8	8	09
=	—	10
cos	8	11
÷	G	12
(	6	13
rcl	5	14
sin	7	15
)	6	16
÷	G	17
#	3	18
6	6	19
·	A	20
2	2	21
8	8	22
3	3	23
1	1	24
9	9	25
÷	G	26
stop	0	27
X	·	28
stop	0	29
#	3	30
3	3	31
7	7	32
7	7	33
=	—	34
stop	0	35

FOURIER ANALYSIS

The Fourier series expansion of the function $f(\omega t)$ is:

$$f(\omega t) = \frac{a_o}{2} + \sum_{k=1}^{\infty}(a_k\cos k\omega t + b_k\sin k\omega t)$$

$$\text{where } a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\omega t)\cos k\omega t\, d(\omega t),$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\omega t)\sin \omega t\, d(\omega t)$$

If $e(\omega t)$ is a periodic voltage of amplitude E_{pk}, its Fourier series is:

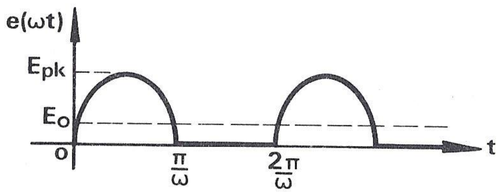
$$e(\omega t) = E_o + \sum_{k=1}^{\infty} E_k\cos(k\omega t + \phi_k) = E_{pk}f(\omega t)$$

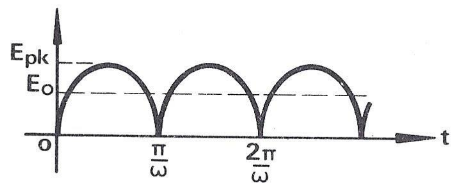
$$\text{where } E_o = \frac{a_o}{2}E_{pk}, \qquad E_k = \sqrt{a_k^2 + b_k^2}\,E_{pk}$$

The coefficients can be formed by numerical integration for non-analysis waveforms.

FOURIER ANALYSIS

Half-wave rectified and full-wave rectified sine wave





Half-wave:

$$e(\omega t) = \frac{1}{\pi} E_{pk} + \frac{E_{pk} \sin \omega t}{2} - \frac{E_{pk}}{\pi} \times \sum_{n=1}^{\infty} \frac{1}{(4n^2-1)\pi} \cos 2n\,\omega t$$

Full-wave:

$$e(\omega t) = \frac{2}{\pi} E_{pk} - \frac{2}{\pi} E_{pk} \sum_{n=1}^{\infty} \frac{1}{(4n^2-1)\pi} \cos 2n\,\omega t$$

÷	G	00
#	3	01
2	2	02
=	—	03
stop	0	04
÷	G	05
#	3	06
1	1	07
·	A	08
5	5	09
7	7	10
0	0	11
7	7	12
9	9	13
6	6	14
3	3	15
=	—	16
sto	2	17
(	6	18
stop	0	19
+	E	20
×	·	21
—	F	22
#	3	23
1	1	24
÷	G	25
)	6	26
×	·	27
rcl	5	28
=	—	29
▼	A	30
goto	2	31
1	1	32
8	8	33
		34
		35

Half-wave:

$E_o = \frac{1}{\pi} E_{pk}$ $E_1 = \frac{E_{pk}}{2}$

$E_{2n} = \frac{E_{pk}}{(4n^2-1)\pi}$ $E_{2n+1} = 0$

Full-wave:

$E_o = \frac{2}{\pi} E_{pk}$ $E_1 = 0$

$E_{2n} = \frac{2E_{pk}}{(4n^2-1)\pi}$ $E_{2n+1} = 0$

Execution:

Half-wave:

E_{pk} / RUN / E_1 / RUN / E_o / 1 / RUN / E_2 / 2 / RUN / E_4 / · · ·

Before re-execution: ▲▼ / ▲▼ / goto / 0 / 0

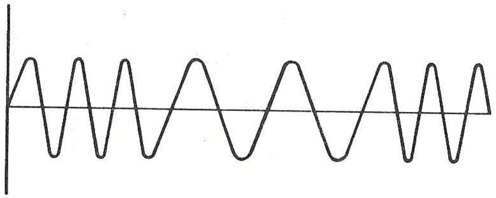
Full-wave:

▲▼ / ▲▼ / goto / 0 / 5 / E_{pk} / RUN / E_o / 1 / RUN / E_2 / 2 / RUN / E_4 / · · ·

FOURIER ANALYSIS

Frequency modulated wave

(iterative computation of Bessel functions)



Where m = modulation index

$e(\omega t) = E_{pk} \cos(\omega_c + m \cos \omega_s)t$

$= E_{pk} J_o(m) \cos \omega_c t +$

$J_1(m)\ [\sin(\omega_c - \omega_s)t - \sin(\omega_c + \omega_s)t]\ -$

$J_2(m)\ [\cos(\omega_c - 2\omega_s)t + \cos(\omega_c + 2\omega_s)t]\ -$

$J_3(m)\ [\sin(\omega_c - 3\omega_s)t - \sin(\omega_c + 3\omega_s)t]\ +$

$J_4(m)\ [\cos(\omega_c + 4\omega_s)t + \cos(\omega_c - 4\omega_s)t]\ + \cdots$

$$\text{where } J_n(m) = \left(\frac{m}{2}\right)^n \sum_{r=0}^{\infty} \frac{(-1)^r}{r!(n+r)!}\left(\frac{m}{2}\right)^{2r}$$

$$= \frac{1}{n!}\left(\frac{m}{2}\right)^n \sum_{r=0}^{\infty} \frac{(-1)^r n!}{r!(n+r)!}\left(\frac{m}{2}\right)^{2r}$$

$$= \frac{1}{n!}\left(\frac{m}{2}\right)^n \lim_{k\to\infty} S_k$$

(where S_k is the sum of the series to k terms)

÷	G	00
#	3	01
2	2	02
=	–	03
ln	4	04
X	·	05
stop	0	06
=	–	07
▼	A	08
e^x	4	09
sto	2	10
÷	G	11
stop	0	12
÷	G	13
(	6	14
stop	0	15
+	E	16
+	E	17
)	6	18
X	·	19
(	6	20
stop	0	21
X	·	22
)	6	23
–	F	24
+	E	25
▼	A	26
MEx	5	27
=	–	28
stop	0	29
▼	A	30
MEx	5	31
▼	A	32
goto	2	33
1	1	34
1	1	35

Execution:

▲▼ / ▲▼ / goto / 0 / 0 / m / RUN / n / RUN /
1 / RUN / n / + / 1 / RUN / m / RUN / S_1
/ RUN / 2 / RUN / n / + / 2 / RUN / m / RUN /
S_2 · · ·
· · · / RUN / r / RUN / n / + / r / RUN / m /
RUN / S_r

(Continue until S_r is sufficiently close to S_{r-1} to have converged to required accuracy.)

Post execution:

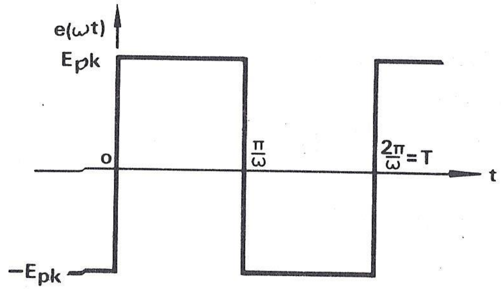
/ ÷ / n! / = / $J_n(m)$

or

/ ÷ / n / ÷ / n – 1 / ÷ / n – 2 / ÷ / · · · / ÷ / 2 / = / $J_n(m)$

FOURIER ANALYSIS

Square wave



$$e(\omega t) = E_{pk} \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin(2n-1)\omega t$$

i.e. $E_K = 0$ if $k = 2n$

$$= \frac{4E_{pk}}{(2n-1)\pi} \text{ if } k = 2n-1$$

Execution:

RUN / E_{pk} / RUN / E_1 / RUN / E_3 / RUN / · · · / RUN / E_{2n-1} / · · ·

If E_{pk} is not entered, the relative amplitude will be given.

Check:

/ ▲▼ / rcl / recovers the current value of (2n – 1). Clear before running again.

Before re-execution:

▲▼ / ▲▼ / goto / 0 / 0

#	3	00
1	1	01
=	–	02
sto	2	03
stop	0	04
X	·	05
#	3	06
1	1	07
·	A	08
2	2	09
7	7	10
3	3	11
2	2	12
3	3	13
9	9	14
5	5	15
=	–	16
stop	0	17
X	·	18
rcl	5	19
÷	G	20
(	6	21
rcl	5	22
+	E	23
#	3	24
2	2	25
=	–	26
sto	2	27
)	6	28
=	–	29
▼	A	30
goto	2	31
1	1	32
7	7	33
		34
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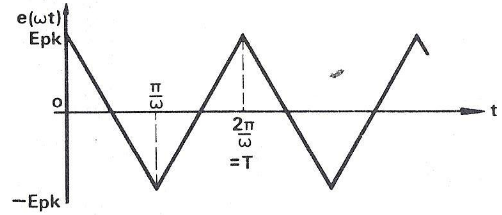
FOURIER ANALYSIS

Triangular wave



$$e(\omega t) = E_{pk} \sum_{n=1}^{\infty} \frac{8}{(2n-1)^2\pi^2} \cos(2n-1)\omega t$$

$$E_k = E_{pk} \frac{8}{(2n-1)^2\pi^2} \quad \text{if } k = 2n-1$$

$= 0$ if $k = 2n$

Execution:

RUN / E_{pk} / RUN / E_1 / RUN / E_3 / · · ·

Post-execution at any stage:

▲▼ / rcl / (2n – 1) / C/CE / (E_{2n-1})

Before execution:

▲▼ / ▲▼ / goto / 0 / 0

#	3	00
1	1	01
=	–	02
sto	2	03
stop	0	04
÷	G	05
#	3	06
1	1	07
·	A	08
2	2	09
3	3	10
3	3	11
7	7	12
=	–	13
stop	0	14
X	·	15
(	6	16
rcl	5	17
X	·	18
)	6	19
÷	G	20
(	6	21
rcl	5	22
+	E	23
#	3	24
2	2	25
X	·	26
sto	2	27
)	6	28
=	–	29
▼	A	30
goto	2	31
1	1	32
4	4	33
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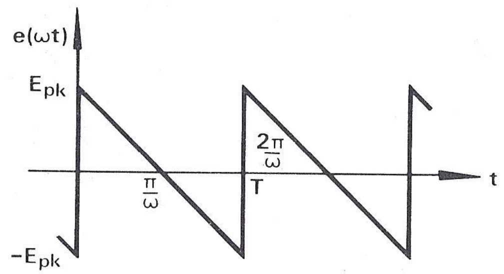
FOURIER ANALYSIS

Sawtooth wave



$$e(\omega t) = E_{pk} \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\omega t$$

$$E_o = 0 \qquad E_n = \frac{2}{n\pi}$$

Execution:

RUN / E_{pk} / RUN / E_1 / RUN / E_2 / RUN / · · · / RUN / E_n · · ·

At any stage, current harmonic order n can be recalled:

/ ▲▼ / rcl / (n) / C/CE / (E_n) / RUN / · · ·

Before re-execution:

▲▼ / ▲▼ / goto / 0 / 0

#	3	00
1	1	01
=	–	02
sto	2	03
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7	7	10
0	0	11
7	7	12
9	9	13
6	6	14
3	3	15
=	–	16
stop	0	17
X	·	18
rcl	5	19
÷	G	20
(	6	21
rcl	5	22
+	E	23
#	3	24
1	1	25
=	–	26
sto	2	27
)	6	28
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▼	A	30
goto	2	31
1	1	32
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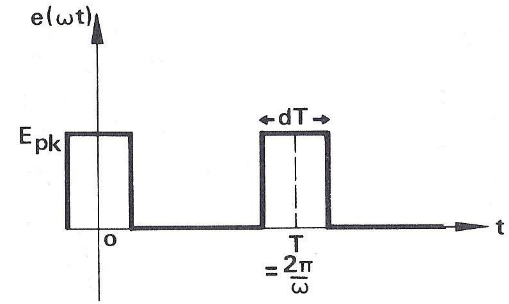
FOURIER ANALYSIS

Rectangular pulse train of duty cycle d



$$e(\omega t) = d\,E_{pk} + E_{pk} \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\pi d \cos n\omega t$$

$$E_o = dE_{pk} \qquad E_n = \frac{2}{n\pi} \sin n\pi d\, E_{pk}$$

Pre-execution:

1·5707963 / ▲▼ / sto / ▲▼ / ▲▼ / goto / 0 / 0 / d / X / E_{pk} / = / E_o

Execution:

n / RUN / d / RUN / E_{pk} / RUN / E_n

n = 1, 2, 3, · · ·

Notes:

Ignore negative signs in results

If E appears after second / RUN / :

(i) Note result r

(ii) Press / 3 / C/CE /

(iii) Enter r / X / E_{pk} / RUN / E_n

X	·	00
rcl	5	01
÷	G	02
(	6	03
+	E	04
X	·	05
stop	0	06
+	E	07
rcl	5	08
–	F	09
+	E	10
rcl	5	11
+	E	12
rcl	5	13
–	F	14
▼	A	15
gin	1	16
0	0	17
9	9	18
+	E	19
rcl	5	20
=	–	21
sin	7	22
)	6	23
÷	G	24
X	·	25
stop	0	26
=	–	27
stop	0	28
▼	A	29
goto	2	30
0	0	31
0	0	32
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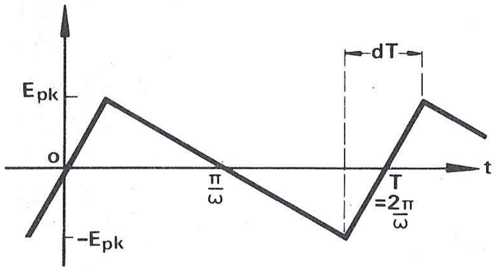
FOURIER ANALYSIS

Asymmetrical triangular wave



$$e(\omega t) = E_{pk} \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2 d(1-d)} \sin n\pi d \sin n\omega t$$

$$E_o = 0 \qquad E_n = \frac{2}{n^2\pi^2 d(1-d)} \sin n\pi d \; E_{pk}$$

Pre-execution:

▲▼ / ▲▼ / goto / 0 / 0 / 1·5707963 / ▲▼ / sto /

Execution:

▲▼ / rcl / X / n / X / d / RUN / d / RUN / E_{pk} / = / E_n

Notes:

Ignore negative signs in results.

If E appears after first / RUN / :

(i) Note the result r

(ii) Press / 3 / C/CE /

(iii) Enter r / X / ▲▼ / (/

(iv) Continue with execution:
d / RUN / E_{pk} / = / E_n

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+	E	05
rcl	5	06
–	F	07
+	E	08
rcl	5	09
+	E	10
rcl	5	11
–	F	12
▼	A	13
gin	1	14
0	0	15
7	7	16
+	E	17
rcl	5	18
=	–	19
sin	7	20
)	6	21
+	E	22
X	·	23
(	6	24
stop	0	25
÷	G	26
–	F	27
#	3	28
1	1	29
=	–	30
)	6	31
÷	G	32
X	·	33
stop	0	34
=	–	35

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